





What if you read a hundred books Or hear a hundred homilies?
Unless your greed is plucked, your anger cease.
What fruit in pouring water for the bath? Our Lord Kūḍala Sañgama laughs to see Born liars whose minds
Are not as good as their words!







What if you read a hundred books Or hear a hundred homilies?
Unless your greed is plucked, your anger cease.
What fruit in pouring water for the bath? Our Lord Kūḍala Sañgama laughs to see Born liars whose minds
Are not as good as their words!







Why, Sir, be angry with those Who're angry with you?
What does it mean to you, Or what their loss?
To show one's anger means
A loss of dignity
To feel it, loss of sense!
The conflagration in your home Unless it burns your house
Does not burn your neighbour's house, O Kūḍala Sañgama Lord?

## DESIGN OF MACHINE ELEMENTS - I

## Subject Code: 15ME54

CREDITS - 04

## Course Objectives

1. Able to understand mechanical design procedure, materials, codes and use of standards.
2. Able to design machine components for static, impact and fatigue strength.
3. Able to design fasteners, shafts, joints, couplings, keys, threaded fasteners riveted joints, welded joints and power screws.

## Course outcomes

## On completion of the course the student will be able to

1. Describe the design process, choose materials.
2. Apply the codes and standards in design process.
3. Analyze the behavior of machine components under static, impact, fatigue loading using failure theories.
4. Design shafts, joints, couplings.
5. Design of riveted and welded joints.
6. Design of threaded fasteners and power screws

## Module-1

Introduction: Fundamentals of Mechanical Engineering Design Mechanical engineering design, Phases of design process, Design considerations, Engineering Materials and their Mechanical properties, Standards and Codes, Factor of safety, Material selection.

Static Stresses: Static loads .Normal, Bending, Shear and Combined stresses. Stress concentration and determination of stress concentration factor.

## THEORY QUESTIONS:

1. What is mechanical engineering design? Explain.
(June/July 2019)( 04 M)
2. Briefly explain the important mechanical properties of metals. Explain each of the briefly.
(Dec 2012) ( 10 Marks) (Dec 2011) (06 Marks)
3. Draw the stress - strain diagrams for a ductile material and a brittle material and show the silent points on them.
(June/July 2014)( 06Marks) (July 2006)( 06Marks)
4. Draw the stress - strain diagrams for a mild steel and a cast iron and show the silent points on them.
(Dec 2012) (06 Marks)
5. Draw the stress - strain diagrams for mild steel subjected to tension. Explain the significance of silent points.
(June/July 2011)( 06Marks)
6. Sketch and explain, biaxial and triaxial stresses, stress tensor and principle stress.
(May 2017) (May/June 2010) (06 Marks) (Dec 2010) ( 08 Marks)
7. What is stress tensor? Explain with neat diagram triaxial stress system and stress, strain relationship for this system.
(Dec.08/Jan. 09(08 Marks)
8. Briefly discuss three dimensional stress field \& stress tensor?
(Dec.14/Jan 2015)( 10Marks)
9. Write briefly note on general procedure used in design. (June/July 2009)
10. What is mechanical engineering design? Explain the steps involved in design with a block diagram.
(May
2017) 
11. Explain the phases of design with neat flow diagram. (June/July 2016)
12. Briefly discuss the factors influencing the selection of suitable material for machine element. (Dec.15/Jan 16) (June/July 2013) ( 05 Marks) (May/June 2010) ( 04 Marks) (Dec.07/Jan. 08(05 Marks)
13. What are the basic requirements of machine elements? Explain briefly.
(June/July 2013)( 05 Marks)
14. Identify the following engineering materials giving specifications:
i) FG350 ii) $\operatorname{Fe} \mathbf{E 3 0 0}$ iii) $\mathbf{C 3 5 M n 7 5}$ iv) $\boldsymbol{X} \mathbf{2 0 C r} \mathbf{1 8 N i 2}$
(June 2012) (04 Marks)
15. Identify the following materials from their designation and indicate the composition: X10Cr18Ni9S3 steel SG400/12 C L C35Mn75 Steel (June/July 2017) (03 Marks)
16. Define standards and codes. (June/July 2016) (Dec 2011) (04 Marks)
17. What is standardization? What are advantages of standardization? State the standards used in machine design. (June/July 2009) (Dec 2012) ( 06Marks)
18. Explain briefly the selection of factor of safety in engineering design.
(Dec.15/Jan 16) (03 Marks)

## INCLINED PLANE STRESS:

## NUMERICAL PROBLEMS:

1. A bar of cross section $25 \mathrm{~mm} \times 25 \mathrm{~mm}$ is subjected to an axial pull of 20 kN . Calculate the normal stress and shear stress on an inclined plane which makes an angle of $60^{\circ}$ with the vertical plane. Also determine the maximum shear stress.
2. A bar of cross section $25 \mathrm{~mm} \times 25 \mathrm{~mm}$ is subjected to an axial pull of 20 kN . Calculate the normal stress and shear stress on an inclined plane which makes an angle of $30^{0}$ with the axis of bar. Also determine the maximum shear stress.
(Dec.08/Jan.09)(08 marks)
3. A tension member is formed, by connecting two wooden scantlings with glue, each $50 \mathrm{~mm} \times 100 \mathrm{~mm}$ at their ends, which are cut at an angle of $60^{\circ}$ as shown in fig. 1.1 the member is subjected to pull F. calculate the safe glue of F. if the permeable normal and shear stress in the glue use are $3 M P a$ and 2 MPa respectively.
4. A machine member is subjected to the following stresses. Determine $\sigma_{x}=90 M P a, \sigma_{y}=60 M P a$.
(i) Normal and shear stresses along a plane inclined at $30^{\circ}$,
(ii) Principal stresses and (iii) Maximum shear stress.
5. A machine member is subjected to the following stresses.

Determine:

$$
\sigma_{\mathrm{x}}=90 \mathrm{MPa} \text { and } \tau_{\mathrm{xy}}=25 \mathrm{MPa} .
$$

(i) Normal and shear stresses along a plane inclined at $30^{\circ}$.
(ii) Principal stresses and max Shear stress.
6. A element is subjected to pure shear stress of $60 M P a$. Find the normal and shear stresses acting on plane inclined at $25^{\circ}$. Also find the principal stresses and maximum shear stress. A state of stress at a point in a structural member is shown in a fig. the tensile principal stress is known to be $84 \mathrm{~N} / \mathrm{mm}^{2}$. Determine
i) The maximum shearing stress at the point and orientation of its plane.
ii) ii) The shearing stress $\tau_{x y}$.

7. Stresses in a two dimensional stressed body as in the Fig, determine:
i. Principal stresses and their direction
ii. Maximum shear stress and their planes. (June/July 2009) (10 Marks)

their location.
(June/July 2014)(06 Marks)

9. An element is acted upon by the following stresses.

$$
\sigma_{x}=120 M P a, \quad \sigma_{y}=90 M P a, \tau_{x y}=30 M P a
$$

i). Compute the stresses on a plane inclined at $20^{\circ}$.
ii). Find principal stresses and their location,
iii). Find maximum shear stress and its location.
10. a point in a structural member is subjected to plane state of stress as shown in fig. determine the following:
i). Normal and tangential stress intensities at an angle of $\varnothing=45^{\circ}$.
ii). Principal stresses $\sigma_{1}$ and $\sigma_{2}$ and their direction.
iii). Maximum shear stress and their direction.

11. A point in a structural member is subjected to plane state of stress as shown in fig. determine the following:
i). Normal and tangential stress intensities at an angle of $\varnothing=45^{\circ}$.
ii). Principal stresses $\sigma_{1}$ and $\sigma_{2}$ and their direction.
iii). Maximum shear stress and their direction.
(Dec.16Jan 17) (Dec.15/Jan 16) (07 Marks)


## INTRODUCTION

"Engineering design is not an art or skill; it is a cognitive, or intellectual, process based on knowledge". John R. Dixon

Engineering design is a purposeful activity directed towards the goal of fulfilling human needs, particularly those which can be met by the technological factors of our culture.

Design is an iterative-creative decision making process directed towards the fulfillment of human needs.


## MACHINE

A machine is a device, a formulation of principles, comprising of various elements arranged together, so as to carry out the needs of a human being
****Write briefly note on general procedure used in design.
What is mechanical engineering design? Explain the steps involved in design

The complete phases of design process are explained as given in Figure 1.2.


Fig. 1.2. Phases in Design

## - Recognition and identification of Need

design begins when an engineer recognizes a need and decides to do something about it. Recognizing the need and identifying it, often requires high creativity because the need is often not obvious.

## - Definition of Problem

Defining the problem means giving a detailed description of the object to be designed, such as its characteristic specifications, dimensions, its inputs and outputs and also all the limitations on these quantities.

## - Synthesis

Synthesis includes the combination of the possible system elements to form a connected whole system.

## - Analysis and Optimization

One may synthesize several components of the system, analyze and optimize them, and return to synthesis to sec what effect this has on the remaining parts of the system.

## - Evaluation

It involves a series of tests that the system is made to undergo to evaluate its characteristics. These include the reliability of the system, its performance when compared to other similar systems.

## - Presentation

This is the final and the most vital phase in the design process. Presentation means communicating the outcome of the evaluation process.

We can classify design as

| Clothing design | Interior design | Highway design |
| :--- | :--- | :--- |
| Landscape design | Building design | Ship design |
| Bridge design | Computer aided design | Heating system design |
| Machine design | Engineering design | Process design |

In fact, this list is endless, since we can also design according to the particular article or product or according to the professional field.

## Material Selection:

What are the basic requirements of machine elements? Explain briefly. (June/July 2013)( 05 Marks)

## What are the factors to be considered for selection of material for a machine component? <br> (Dec.16/Jan. 17)(05 M)

The selection of a material is one of the important decisions taken by the designer. in general, the major factors governing the selection of material are:
a). Cost : The cost of the component is generally dictated by the material cost and the processing cost.
b). Availability: Materials should be avoided and use should be made of those materials which are openly available in die market.
c). Manufacturing: In certain cases, components are to be designed in a particular form and the manufacturing process is dictated by the shape of the component.
d). Mechanical Properties: Engineering applications require the use of many important properties from a material such as strength (static and under cyclic conditions), plasticity, rigidity, elasticity, ductility, malleability, hardness, toughness, shock resistance, wear resistance, creep and temperature characteristics, corrosion resistance and frictional properties.

Mode of Failure:
Different components show different modes of failure, which in turn would be dependent on the kind of loading and the type of material used. Failure does not necessarily mean breakage. A failure is said to happen when the machine clement is unable to perform its function satisfactorily. Failure can happen by :
a). Elastic Deflection : In this case the defection within an element happens within elastic limits. However, it is possible that with this deflection, the element may not serve its intended function. For example, a shaft having a large lateral deflection would result in improper meshing of the gears mounted on it.
b). Yielding : If the strain on the component exceeds the elastic limit, plastic deformation sets in. This is called as yielding and is observed in ductile materials. A component under this situation is said to have failed.
c). Fracture: In brittle materials like cast iron, yielding is not observed. It is seen that the material fails by fracture, without any plastic deformation.

## *** Briefly discuss the factors influencing the selection of suitable material for machine element. (June/July2013)(May/June2010)(Dec.07/Jan. 08)(05 M) <br> DESIGN CONSIDERATION

During the designing process we have to consider certain characteristics that influence the design of the element or system to be designed. Some important ones are listed below:

| Strength | Life | volume | Reliability | Thermal properties |
| :--- | :--- | :--- | :--- | :--- |
| Wear | Friction | Processing | Safety | Corrosion |
| Utility | Size | Control | Styling | Liability |

Define standards and codes.
(Dec.16/Jan. 17)(Dec 2011) (04 Marks)
What is standardization? What are advantages of standardization? State the standards used in machine design.
(June/July 2009)(06 Marks)(Dec 2012)(4 M)

## USE OF STANDARDS

A standard is a set of specifications for parts, materials or processes, intended to achieve uniformity, efficiency and specified quality.

As a summary, standards prove useful for the following reasons:

1. Interchangeability of Parts: An example of interchangeability can be easily understood if we consider, a Hexagonal Bolt M10 x 40L of any company, which can easily be interchanged with that of the other, as both the companies manufacture to the same standards.
2. Easy Replacements: As size relations are specified and limited, the components can be stocked by dealers, thus resulting in easy replacements.
3. Reduced Stock or Inventory: Easy replacements from dealers reduce in-house inventory, which in turn reduce the inventory carrying costs.
4. Better Efficiency \& Specified Quality: The indirect guarantee and expertise of the standard is passed to the user, thus reducing his design costs, with good assurance of required quality.
5. Mass Production of Components: With standardization, mass production of components is easier. Different components can also be manufactured at various plants and assembled together without affecting the assembly.

## CODES

A code is a set of specifications for the analysis, design, and manufacture and construction of something. The purpose of a code is to achieve a specified degree of safety, efficiency and performances or quality.

All of the organisations and societies listed below have established specifications for standards and safety or design codes.

The name of organisation provides a clue to the nature of the standard or code.
The organisations of interest to Mechanical Engineers are

1. Aluminium Association (AA)
2. American Gear Manufactures Association (AGMA)
3. American Institute of Steel Construction (AISC)
4. American Iron and Steel Institute (AISI)
5. American National Standards Institute ANSI)
6. American Society for Metals (ASM)
7. American Society of Mechanical Engineers (ASME)
8. American Society of Testing and Materials (ASTM)
9. American Welding Society (AWS)
10. Anti-Friction Bearing Manufactures Association (AFBMA)
11. British Standards Institution (BSI)
12. British Standards Association (BSA)
13. Bureau of Indian Standards (BIS)
14. Bureau of International Weights and Measures (BIWM)
15. German National Standards (DIN)
16. Industrial Fasteners Institute (IFI)
17. Institution of Mechanical Engineers (I.Mech. E)
18. International Bureau of Weights and Measures (BIPM)
19. International Standards Organisation (ISO)
20. National Bureau of Standards (NBS)
21. Society of Automotive Engineers
(SAE)

Besides these national and international standards, each company within itself may have its own company standards, whose specifications and rulings hold true in each of their plants.

## PROPERTIES OF MATERIALS

The properties of materials used in engineering design may be classified as:

- Physical and chemical properties: Chemical composition, structure, homogeneity, specific weight, melting point temperature, thermal conductivity, and coefficient of thermal expansion.
- Mechanical properties:

Elastic limit, modulus of elasticity, strength in tension, compression, bending and torsion, endurance limit, hardness, and wear resistance.

- Technological properties: Forgeability, castability, malleability, bending, machinability, weldability, and the like.

Briefly explain the important mechanical properties of metals. Explain each of the briefly. (Dec 2012) (10 Marks) (Dec 2011) (06 Marks)

## Mechanical Properties

The mechanical properties are those which indicate how the material would behave when subjected to various types of loads. These properties are established by various tests. Standardized test methods are described in various standards. A good knowledge of mechanical properties permits the designer to determine the size and shape of the machine components.

The most commonly evaluated mechanical properties are discussed below:
Isotropy: A material that displays the same elastic property in all direction is termed Isotropic.

Elasticity: It is a property of the material by virtue of which a machine component can regain its original shape and size when the external load acting upon it is removed. The modulus of elasticity is an index for evaluation of elasticity.


Strength: The strength of a member is defined as the ability of its material to withstand yielding and/or fracture against applied forces so that it can continue to perform its designed function in the machine. The stress magnitude which corresponds to permanent deformation of a definite amount, usually up to 0.2 per cent of original gauge length, is termed yield strength ( $\sigma_{y}$ ).

## Draw the stress - strain diagrams for a ductile material and a brittle material and show the silent points on them. <br> (Dec.16/Jan. 17) (June/July 2014)(06 Marks) (July 2006)(06 Marks)

Draw the stress - strain diagrams for a mild steel and a cast iron and show the silent points on them.
(Dec 2012) (06 Marks)
Draw the stress - strain diagrams for mild steel subjected to tension. Explain the significance of silent points. (June/July 2011)(06 Marks)

Ductility: It is a properly of the material by virtue of which it can undergo a large permanent deformation in tension before being fractured, it is usually expressed as percent elongation in 50 mm gauge length.


Stress - Strain characteristric of mild steel (Ductile Material)


Brittleness: It is a characteristic opposite to ductility. Lack of plasticity is known as brittleness. A material may be considered brittle if its elongation prior to rupture in tension is less than 5 per cent in a 50 mm long specimen.

Malleability: It is a property of the material which permits large plastic deformation in compression without fracture. In other words, a malleable material is the one that can easily be flattened or rolled.

Hardness: It is the ability of the material to resist penetration, plastic indentation, abrasion, or scratching. It is also an indicator of wear resistance under certain conditions. It is usually expressed by numbers which are dependent upon the methods of testing. Mostly, two popular methods, called the Brinell and Rockwell hardness numbers, are in use.

The Rockwell $\left(R_{c}\right)$ and Brinell hardness number $(\mathrm{BHN})$ are related by the following relation:

$$
R_{C}=88(B H N)^{0.162}-192
$$

Experimental results have shown that the ultimate tensile strength and the surface endurance strength of plain carbon steel are related by the following: $\sigma_{U t} \approx 3.45 B H N N / \mathrm{mm}^{2} \quad ; \quad \sigma_{\text {ens }} \approx(2.75 B H N-70) \mathrm{N} / \mathrm{mm}^{2}$

The hardness of synthetic materials like rubber, foam, plastic, etc. can be measured by the durometer. The durometer hardness number is based on any arbitrary scale of $0-100$.

Resilience: The ability of a material to absorb energy within its elastic limit without any permanent deformation is called resilience.

Toughness: The ability of a material to absorb energy before it fractures is called toughness. The Bureau of Indian Standards (BIS) has suggested two test procedures to measure the toughness. They are (i) Charpy impact test and (ii) Izod impact test.


Creep: A material under a heavy steady load at a high temperature for a long period begins to deform plastically. This deformation, called creep, is time dependent and increases with time until fracture lakes place.

Wear: $\quad$ The material on the surface keeps getting dislodged in the form of small particles because of relative motion between mating surfaces. This type of depletion of the material is called wear. Wear is attributed to several phenomena, namely:
(a) Scuffing-This type of wear is caused owing to failure of the lubricant film which creates rnicrowelds between surface asperities. These microwelds are sheared by relative motion between sliding parts, thereby dislodging small particles from surfaces. This type of wear is also called galling or seizing.
(b) Abrasion-When hard foreign particles such as dust, metal grit or metal oxides find their way in between two rubbing surfaces, they cause abrasive wear,
(c) Pitting-The wear caused by cyclic contact stresses between mating surfaces is called pitting,
(d) Frettage-A surface damage caused by small movements between mating surfaces is known as frettage.

## Technological Properties

Technological properties are those which relate to manufacturing of a machine component, namely machinability, formability, castability, etc. A designer must also have a thorough knowledge of these properties because the material selected for a component should be such that it can easily be manufactured by using the available facilities.

Machinability: The term machinability is used to specify the relative ease with which a given metal can be machined. The machinability rating of a material is usually decided on the basis of the following parameters: (i) power required, (ii) tool-life, (iii) machining time, and (iv) surface quality. Machinability of a metal depends upon its (a) hardness, (b) strength, and (c) chemical composition. The presence of sulphur, lead, and manganese in steel improves its machinability.

Formability: It is an indication of suitability of a metal for a machine part that requires forming. Ductility and tensile strength are important properties which determine the formability of a metal.

Castability: The term castability is an indication of how easy it is to cast The important factors which affect castability are: (i) solidification temperature, (ii) amount of metal shrinkage after solidifying, and (iii) the strength at the temperature just below which the solidification starts.

Weldability: The term weldability is used to indicate the ease with which a successful weld can be made.

Forgeability: It is an indication of ease involved in forging a machine component. In forging, a larger plastic range is desirable because it allows a longer time to work with the metal before reheating becomes necessary.

## BSI system of designation:

| Elements | Multiplying Factor |
| :--- | :--- |
| $\mathrm{Cr}, \mathrm{Co}, \mathrm{Ni}, \mathrm{Mn}, \mathrm{Si}$ and W | 4 |
| $\mathrm{Al}, \mathrm{Be}, \mathrm{V}, \mathrm{Pb}, \mathrm{Cu}, \mathrm{Nb}, \mathrm{Ti}, \mathrm{Ta}, \mathrm{Zr}$ and Mo | 10 |
| $\mathrm{C}, \mathrm{P}, \mathrm{S}, \mathrm{N}$ | 100 |

Identify the following engineering materials giving specifications:
i) FG350
ii) FeE300
iii) C35Mn75
iv) $\mathbf{x 2 0 C r} 18 \mathrm{Ni} 2$

Ans:
(June 2012) (04 Marks)
i) FG350 - Grey Iron casting

350 - Minimum tensile strength in $\mathbf{N} / \boldsymbol{m m}^{\mathbf{2}}\left(\sigma_{\boldsymbol{u}}=\mathbf{3 5 0} \mathbf{N} / \boldsymbol{m m}^{\mathbf{2}}\right)$
ii) FeE300 - Semi killed steel

$$
350 \text { - Minimum yield strength }\left(\sigma_{y}=\mathbf{3 0 0}^{\mathbf{N}} / \boldsymbol{m m}^{\mathbf{2}}\right)
$$

iii) C 35 Mn 75

C35 indicates carbon as ten times minimum limit (total carbon $=0.35 \%$ )
Manganese 7.5\%
iv) X 20 Cr 18 Ni 2 - steel with 0.20 \% Carbon
$0.18 \%$ Chromium $\quad 0.02 \%$ Nickel $($ Page -411$)$
19. Identify the following materials from their designation and indicate the composition: X10Cr18Ni9S3 steel SG400/12 C L C35Mn75 Steel (June/July 2017) (03 Marks)

X10Cr18Ni9S3 steel - Steel with 0.10 \% Carbon
$0.18 \%$ Chromium $\quad 0.9 \%$ Nickel (Page - 411)
C35Mn75 Steel - Steel with C35 indicates carbon as ten times minimum limit (total carbon $=3.5 \%$ )

Mn75 - indicates manganese as four times minimum limit (total manganese $=18$ \%)(Page - 412)

SG400/12 C L - Spheroidal graphite iron (Page - 411)

$$
400 \text { - Minimum tensile strength }\left(\sigma_{u}=400 \mathrm{~N} / \mathbf{m m}^{2}\right)
$$

## STRESSES AND STRAINS

## Loads and Forces

External forces acting on a rigid body are termed as loads. These loads may arise due to dead loads of members, live loads, wind loads, earthquake effects, fluid pressures, support settlements, frictional resistance, etc.
i) Dead or Steady Load: a load is said to be a dead or steady load when it does not change in magnitude or direction.
ii) Live or variable load: a load is said to be live or variable load when it changes continually.
iii) Suddenly applied or Shock load: a load is said to be a suddenly applied or shock load when it is suddenly applied or removed.
iv) Impact load: a load is said to be an impact load when it is applied with some initial velocity.

## Tensile Stress ( $\boldsymbol{\sigma}_{\boldsymbol{t}}$ )

The intensity of the tensile stress is given by

$$
\sigma_{t}=\frac{\mathrm{R}}{\mathrm{~A}}=\frac{\mathrm{P}}{\mathrm{~A}}=\frac{\mathrm{F}}{\mathrm{~A}} \quad \operatorname{Eqn}(\mathbf{1 . 1}) \mathbf{P}-\mathbf{2}
$$



Compressive Stress ( $\boldsymbol{\sigma}_{\boldsymbol{c}}$ )
The intensity of the compressive stress is given by

$$
\begin{equation*}
\sigma_{c}=\frac{\mathrm{R}}{\mathrm{~A}}=\frac{\mathrm{P}}{\mathrm{~A}}=\frac{\mathrm{F}}{\mathrm{~A}} \tag{Eqn}
\end{equation*}
$$



The intensity of the shear stress is given by
$\boldsymbol{\tau}=\frac{\mathrm{R}}{\mathrm{A}}=\frac{\mathrm{P}}{\mathrm{A}}=\frac{\mathrm{F}_{\mathrm{s}}}{\mathrm{A}_{\mathrm{s}}} \quad \operatorname{Eqn}(1.4) \mathrm{P}-2 \quad$ Shear stress (in double shear) $\boldsymbol{\tau}=\frac{\mathrm{P}}{2 \mathrm{~A}}$

(a) Single shear. (b) Double shear.

## Bearing Stress $\sigma_{b}\left(\sigma_{c}\right)$

A localized compressive stress at the surface of contact between two members of a machine parts that are relatively at rest is known as bearing stress or crushing stress. the bearing stress or crushing stress (stress at the surface of contact between the rivet and a plate),

$$
\sigma_{b}\left(\sigma_{c}\right)=\frac{P}{\text { d.t.n }}
$$

Where, $\quad d=$ diameter of the rivet, $d . t=$ projected area of the rivet
$t=$ thickness of the plate, $n=$ number of rivets per pitch length in bearing or crushing.


Fig. 1.11.5. Bearing stress in a revited joint.

A bearing stress is given by

$$
p_{b}=\frac{P}{l . d}
$$

$\operatorname{Eqn}(15.18) P-303$
Where, $\quad p_{b}=$ average bearing pressure, $P=$ radial load on the journal, $l=$ length of the journal in contact, $\quad d=$ diameter of the journal.

## $\operatorname{STRAIN}(\varepsilon)$

Strain is a measure of the deformation produced in a member by the loads.
it is denoted by $\varepsilon$, it is a ratio between change in dimension and original dimension, and as such it has no unit.

$$
\therefore \quad \operatorname{Strain}(\varepsilon)=\frac{\text { change in dimension }}{\text { original dimension }}
$$

SHEAR STRAIN $\left(\boldsymbol{\varepsilon}_{s}\right)$
It is a measure of the angle through which a body is deformed by the applied force. Then,

$$
\text { Shear strain, } \quad \varepsilon_{s}=\frac{c c^{\prime}}{c b}
$$



Fig.1.11.10
Shear strain, $\quad \Phi=\gamma=\frac{\tau}{G} \quad \mathbf{E q n ( 1 . 5 ) P - 2}$

## Volumetric Strain ( $\varepsilon_{v}$ )

The ratio between the change in volume and the original volume is known as volumetric strain. It is denoted by $\varepsilon_{v}$.

$$
\text { Valumetric strain, } \varepsilon_{v}=\frac{\text { change in volume }}{\text { original volume }}=\frac{\delta v}{v}
$$



Volumetric Compression

## ELASTICITY AND ELASTIC LIMIT

A material is said to be perfectly elastic if the deformation produced due to the application of external load disappears completely with the removal of the load. This property of the material is called elasticity.

## Young's Modulus of Elasticity (Table -1.1) P-7

Hooke's law states that within an elastic limit stress is proportional to strain,

$$
\text { stress } \alpha \text { strain } \quad \text { or } \quad \frac{\text { stress }}{\text { strain }}=\mathrm{a} \quad \text { constant }
$$

This constant is known as coefficient of elasticity or Modulus of elasticity. The ratio between tensile stress and tensile strain or compressive stress and compressive strain is called Young's Modulus of Elasticity and is denoted by $E$

$$
\therefore \quad E=\frac{\text { Tensile stress }}{\text { tensile strain }}=\frac{\sigma_{t}}{\varepsilon_{t}} \quad \text { or } \quad E=\frac{\text { compressive stress }}{\text { compressive strain }}=\frac{\sigma_{c}}{\varepsilon_{c}}
$$

| Values of E for the commonly used engineering materials |  |
| :--- | :--- |
| Material | Modulus of Elasticity (E) in GPa |
| Steel \& Nickel | 200 to 220 |
| Wrought iron | 190 to 200 |
| Cast iron | 100 to 160 |
| Copper | 110 to 90 |
| Brass | 90 to 80 |
| Aluminium | 80 to 60 |
| Timber | 10 |

## Shear Modulus or Modulus of Rigidity (Table -1.1) P-7

The ratio between shear stress and shear strain is called Shear Modulus or Modulus of Rigidity. It is denoted by C or $G$ or $N$.

$$
\therefore \quad G(C \text { or } N)=\frac{\text { Shear stress }}{\text { Shear strain }} \frac{\tau}{\emptyset}=\frac{\tau}{\varepsilon_{s}}
$$

| Values of C for the commonly used engineering materials |  |
| :--- | :--- |
| Material | Modulus of Rigidity (C) in GPa |
| Steel | 100 to 80 |
| Wrought iron | 90 to 80 |
| Cast iron | 50 to 40 |
| Copper | 50 to 30 |
| Brass | 50 to 30 |
| Timber | 10 |

The ratio between the normal stress and the volumetric strain is called Bulk modulus of elasticity and is denoted by $K$

$$
\therefore \quad K=\frac{\text { Normal stress }}{\text { volumetric strain }}=\frac{\sigma_{n}}{\varepsilon_{v}}
$$

## Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to torsion. The stress set up by torsion is known as torsional shear stress. It is zero at the centroidal axis and maximum at the outer surface.


Fig. 1.11.7. Torsional Shear Stress

The following equation: $\quad \frac{\tau}{r}=\frac{T}{J}=\frac{G . \theta}{l} \quad \operatorname{Eqn}(\mathbf{1 . 1 5 )} \mathbf{P}-\mathbf{3}$
The Equation (i) is known as torsion equation.
Where $\tau=$ torsional shear stress induced at the outer surface of the shaft or maximum shear stress,
ror $c=$ radius of the shaft, $\quad$ or $M_{t}=$ torque or twisting moment,
$J=$ second moment of area of the section about its polar axis or polar moment of inertia,
$G=$ modulus of rigidity for the shaft material, $l=$ length of the shaft $\theta=$ angle of twist in radius on a length $J=$ Polar Moment of Inertia, $\begin{array}{rlrl}J= & \frac{\pi}{32} d^{4} \text { for Solid circular shaft } & r=\frac{d}{2} & \text { for Solid circular shaft } \\ & J & r=\frac{d_{o}}{2} & \text { for Hollow circular shaft } \\ & =\frac{\pi}{32}\left(d^{4}{ }_{o}\right. & & \\ & \left.-d^{4}{ }_{i}\right) \text { Hollow circular shaft } & & \end{array}$


Hollow Shaft


Solid Shaft
$\therefore \quad$ Shear Stress, $\quad \tau=\frac{M_{t} \cdot r}{J}=\frac{T . c}{J} \quad \operatorname{Eqn}(3.1) \mathbf{P}-42$
Power: $\quad$ Power, $P=\frac{2 \pi n M_{t}}{60000} \mathrm{~kW}$
Where $\mathrm{n}=$ speed in rpm $\mathrm{P}=$ power in $\mathrm{kW} \quad M_{t}=$ torque in $\mathrm{N}-\mathrm{m}$

$$
\begin{gathered}
\therefore \quad M_{t}=\frac{60000 P}{2 \pi n}=\frac{9550 P}{n} N-m \\
T=\frac{9550 \times 10^{3} P}{n}=\frac{9.55 \times 10^{6}(P)}{n} N-m m \operatorname{Eqn}(\mathbf{3 . 3 a}) \mathbf{P}-42
\end{gathered}
$$

## Bending stress

The bending equation is given by

$$
\begin{equation*}
\frac{M}{I}=\frac{\sigma}{r}=\frac{E}{R} \tag{Eqn}
\end{equation*}
$$



Fig.1.11.8. Bending Stress in Straight beams.
Where $\quad \mathrm{M}=$ bending moment acting at the given section, = Force $\mathrm{x} \perp l r$ distance.
$I=$ moment of inertia of the cross - section about the neutral axis,
$r=$ distance from the neutral axis (NA) to the extreme fiber,
$E=$ young's modulus of the material of the beam, and
$R=$ radius of curvature of the beam.

$$
\sigma=\text { bending stress, }
$$

$I=$ Moment of Inertia, $\quad$ (Table-1.3)(P-8)

$$
I=\frac{\pi}{64} d^{4} \quad \text { for Solid circular shaft }
$$

$I=\frac{\pi}{64}\left(d^{4}{ }_{o}-d^{4}{ }_{i}\right)$ for Hollow circular shaft $\quad Z=\frac{I}{C}=$ Section Modulus $r=\frac{d}{2} \quad$ for Solid circular shaft, $\quad r=\frac{d_{o}}{2} \quad$ for Hollow circular shaft

$$
\therefore \quad \text { Bending Stress }=\frac{M_{b} C}{I}=\frac{M_{b}}{(I / C)}=\frac{M_{b}}{Z}
$$



Hollow Shaft


Solid Shaft

## POISSON'S RATIO ( $\boldsymbol{\mu}$ or $\frac{1}{m}$ ) <br> (Table -1.1) P-7

The ratio between the lateral strain and the longitudinal strain is called Poisson's ratio and is denoted by $\boldsymbol{\mu}$ or $\frac{\mathbf{1}}{\boldsymbol{m}}$.
$\therefore \quad$ Poission's ratio, $\boldsymbol{\mu}$ or $\frac{1}{m}=\frac{\text { Lateral strain }}{\text { longitudinal strain }}$

For most metals, $\frac{\mathbf{1}}{\boldsymbol{m}}$ lies between 0.33 and 0.25 .

Values of Poisson's Ratio for the commonly used engineering materials

| Material | Poisson's Ratio $\left(v\right.$ or $\mu$ or $\left.\frac{\mathbf{1}}{\mathbf{m}}\right)$ |
| :--- | :--- |
| Steel | 0.33 to 0.25 |
| Cast iron | 0.27 to 0.23 |
| Copper | 0.34 to 0.31 |
| Brass | 0.42 to 0.32 |
| Aluminium | 0.36 to 0.32 |
| Rubber | 0.50 to 0.45 |

## FACTOR OF SAFETY

The ratio of the ultimate stress and the working stress for a material is called factor of safety.

It is defined, in general, as the ratio of the maximum stress to the working stress. Mathematically,

$$
\text { Factor of Safety }=\frac{\text { Maximum stress }}{\text { Working or design stress }}
$$

Incase of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of Safety is based upon the yield point stress. In such cases,

$$
\text { factor of safety }=\frac{\text { Yield point stress }}{\text { Working or design stress }}
$$

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$
\text { factor of safety }=\frac{\text { Ultimate stress }}{\text { Working or design stress }}
$$

| Values of factor of safety |  |  |  |
| :--- | :--- | :--- | :--- |
| Material | Steady load | Live load | Shock load |
| Cast iron | 5 to 6 | 8 to 12 | 16 to 20 |
| Wrought iron | 4 | 7 | 10 to 15 |
| Steel | 2 to 4 | 8 | 12 to 16 |
| Soft materials and | 6 | 9 | 15 |
| alloy | 9 | 12 | 15 |
| Leather | 7 | 10 to 15 | 20 |
| timber | 7 |  |  |

The factor of safety depends on many considerations as follows:
(i) The nature of loading (ii) The homogeneity of the materials used
(iii) The accuracy with which stresses in members and external forces can be evaluated
(iv) The degree of safety required (v) The degree of economy desired.

When the material is subjected to varying stresses, the factor of safety is high. For most engineering structures a factor of safety between 2 and 6 is suitable.

### 1.12. LOAD AND STRESS

three states of stress in the element which are:
(i) Uni-axial stress
(ii) Bi-axial stress
(iii) Tri-axial stress

Sketch and explain, biaxial and triaxial stresses, stress tensor and principle stress.
(May 2017) (May/June 2010) (06 Marks) (Dec 2010) (08 Marks)

## UniAXIAL STRESS

The unidirectional tensile or compressive stress is calculated from the following
equation:

$$
\sigma=\frac{F}{A}
$$



Fig. 1.12.1. A Bar subjected to Uniaxial stress.
Where $\quad \sigma$ is the unidirectional stress $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$.
$F$ is the applied force ( N )
$A$ is the area of cross-section $\left(\mathrm{mm}^{2}\right)$

When a member is subjected to two equal and opposite forces acting tangentially across the resisting section of the member in such a way that its one layer tends to slide over the other, the member is said to be in the state of shear.
$\tau=\frac{F}{A} \quad$ Where $\quad$ ' $\tau$ ' is the shear stress $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$.


(b) State of shearing

(c) Shear Stresses

Fig. 1.12.2. A Rivet under the state of Shear Forces

## Stresses induced by normal and shear load

Sign conventions:

1. Sign for normal stress: tensile stresses are considered positive and compressive stresses are considered negative as shown below.

2. Sign for shear stress: shear stresses are considered positive when the pair of shear stresses acting on opposite and vertical sides of an element from a counter clockwise couple as shown in below figure.

3. The angle of inclination $\theta$ is considered positive when measured from the vertical plane on which the principal stress $\sigma_{\theta}$ and the tangential shear stress $\tau_{\theta}$ on an inclined plane which makes an angle of $\theta$ to the vertical plane on which the principle stress $\sigma_{\mathrm{x}}$ acts.


## Unidirectional stress

Figure 1.12.3, shows a member subjected to unidirectional stress $=\sigma_{x}$ acting on face AB. (tensile or compressive).

The normal stress on a plane AC inclined at $\theta . \quad \sigma_{n}=\sigma_{x} \cos ^{2} \varnothing \quad$ and
Shear stress acting along plane AC inclined at $\theta . \quad \tau_{n}=\frac{\sigma_{x}}{2} \sin 2 \emptyset$
Max shear stress, $\tau_{\max }=\frac{\sigma_{x}}{2}$ at $\emptyset=45^{0}$


## Biaxial Stress



Fig.1.12.4. A Biaxial Stress Element
Figure 1.12.5, shows an element subjected to biaxial stresses $\sigma_{x}$ along $x$ direction (tensile or compressive) and $\sigma_{y}$ along $y$ direction $\perp l r$ to $x$.

Tensile or compressive Normal stress on the plane AC inclined at $\emptyset$.

$$
\sigma_{n}=\left(\frac{\sigma_{x}+\sigma_{y}}{2}\right)+\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \cos 2 \emptyset+\tau_{x y} \sin 2 \emptyset
$$

Shear stress on the plane AC inclined at $\theta$.

$$
\tau_{n}=\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \sin 2 \emptyset-\tau_{x y} \cos 2 \emptyset \quad \operatorname{Eqn}(1.8) \mathbf{P}-2
$$

## Principal Stresses and Principal Planes

These three mutually perpendicular planes which have no shear stress are known as principal planes and the direct stresses along these planes are known as principal stresses.

The planes on which the maximum shear stress act are known as planes of maximum shear

$$
\begin{aligned}
& \sigma_{1}=\frac{\left(\sigma_{x}+\sigma_{y}\right)}{2}+\sqrt{\left[\frac{\sigma_{x}-\sigma_{y}}{2}\right]^{2}+\left(\tau_{x y}\right)^{2}} \quad \operatorname{Eqn}(1.11 \mathbf{a}) \mathbf{P}-\mathbf{2} \\
& \sigma_{2}=\frac{\left(\sigma_{x}+\sigma_{y}\right)}{2}-\sqrt{\left[\frac{\sigma_{x}-\sigma_{y}}{2}\right]^{2}+\left(\tau_{x y}\right)^{2}} \quad \operatorname{Eqn}(1.11 b) \mathbf{P}-\mathbf{2}
\end{aligned}
$$

The planes of maximum shear stress are at right angles to each other and are inclined at $45^{\circ}$ to the principal planes. The maximum shear stress is given by one-half the algebraic difference between the principal stresses,

Maximum shear stress

$$
\tau_{\max }=\frac{\sigma_{1}-\sigma_{2}}{2}= \pm \sqrt{\left[\frac{\sigma_{x}-\sigma_{y}}{2}\right]^{2}+\left(\tau_{x y}\right)^{2}} \quad \text { at } \emptyset=45^{0} \quad \operatorname{Eqn}(\mathbf{1 . 1 2 )} \mathbf{P}-2
$$

## Unidirectional stress combined with shear stress

Figure 1.27 shows an element acted upon by an unidirectional stress $\sigma_{x}$ (tensile or compressive) along with a shear stress $\tau_{x y}$.

There is no stress acting along $y$ direction $\perp \operatorname{lr}$ to $x$.
Hence, $\sigma_{y}=0$.


The normal stress on a plane AC inclined at $\theta$.

$$
\sigma_{n}=\sigma_{x} \cos ^{2} \varnothing+\tau_{x y} \sin 2 \emptyset \quad \operatorname{Eqn}(1.7) \mathbf{P}-2 \text { and }
$$

Shear stress acting along plane AC inclined at $\varnothing$.

$$
\tau_{n}=\frac{\sigma_{x}}{2} \sin 2 \emptyset+\tau_{x y} \cos 2 \emptyset \quad \quad \operatorname{Eqn}(1.7) \mathbf{P}-2
$$

Maximum principle stress, $\sigma_{1}=\frac{1}{2}\left[\sigma_{x}+\sqrt{\sigma_{x}{ }^{2}+4 \tau^{2}{ }_{x y}}\right] \quad$ Eqn(1.11a)P-2
Minimum principle stress, $\sigma_{2}=\frac{1}{2}\left[\sigma_{x}-\sqrt{\sigma_{x}{ }^{2}+4 \tau^{2} x y}\right]$
$\operatorname{Eqn}(1.11 b) P-2$

Max shear stress, $\tau_{\max }=\frac{1}{2} \sqrt{\sigma_{x}{ }^{2}+4 \tau^{2}{ }_{x y}}$
$\operatorname{Eqn}(1.12) P-2$

Note: Tensile stresses are considered + ve and compressive stresses are regarded as - ve values in all the above equation.

## Pure shear

Figure 1.12.7, shows on element subjected to pure shear stress $\tau_{x y}$.
Normal stress on the plane AC inclined at $\emptyset$.

$$
\begin{aligned}
& \sigma_{n}=\tau_{x y} \sin 2 \emptyset \\
& \tau_{n}=-\tau_{x y} \cos 2 \emptyset
\end{aligned}
$$

Shear stress on the plane AC inclined at $\emptyset$.


Maximum principle stress, $\sigma_{1}=\tau_{x y}$
Minimum principle stress, $\sigma_{2}=-\tau_{x y}$

$$
\text { Maximum shear stress } \quad \tau_{\max }=\frac{\sigma_{1}-\sigma_{2}}{2}=\tau_{x y} . \quad \text { at } \emptyset=45^{0}
$$

## Biaxial stresses combined with shear stress

Figure 1.26 shows an element acted upon by biaxial stresses combined with shear stress.
$\sigma_{x}=$ Normal stress along $x$ direction, tensile or compressive
$\sigma_{y}=$ Normal stress along $y$ direction, tensile or compressive
$\tau_{x y}=$ shear stress on $x y$ plane.


Normal stress on the plane AC inclined at $\emptyset$.

$$
\sigma_{n}=\left(\frac{\sigma_{x}+\sigma_{y}}{2}\right)+\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \cos 2 \emptyset+\tau_{x y} \sin 2 \emptyset \quad \ldots \ldots \ldots \operatorname{Eqn}(\mathbf{1} .7) \mathbf{P}-\mathbf{2}
$$

Shear stress on the plane AC inclined at $\emptyset$.

$$
\tau_{n}=\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \sin 2 \emptyset-\tau_{x y} \cos 2 \emptyset \quad \operatorname{Eqn}(\mathbf{1 . 8}) \mathbf{P}-\mathbf{2}
$$

Maximum \& Minimum principle stress,

$$
\sigma_{1,2}=\frac{\left(\sigma_{x}+\sigma_{y}\right)}{2} \pm \sqrt{\left[\frac{\sigma_{x}-\sigma_{y}}{2}\right]^{2}+\left(\tau_{x y}\right)^{2}}
$$

$\operatorname{Eqn}(1.11 \mathrm{a} \& b) \mathrm{P}-2$
Maximum Shear stress,
$\tau_{\max }=\frac{\sigma_{1}-\sigma_{2}}{2}= \pm \sqrt{\left[\frac{\sigma_{x}-\sigma_{y}}{2}\right]^{2}+\left(\tau_{x y}\right)^{2}}$

$$
\operatorname{Eqn}(1.12) P-2
$$

The angle of principle planes, $\operatorname{Tan} 2 \emptyset=\frac{2 \tau_{x y}}{\sigma_{x}-\sigma_{y}}$

$$
\emptyset_{1,2}=\frac{1}{2} \tan ^{-1}\left[\frac{2 \tau_{x y}}{\sigma_{x}-\sigma_{y}}\right] \quad \emptyset_{1} \text { and } \emptyset_{2} \text { are at } 90^{\circ} \text { apart. }
$$

The angle of max shear planes, $\operatorname{Tan} 2 \emptyset_{s}=\frac{\sigma_{x}-\sigma_{y}}{2 \tau_{x y}}$
$\operatorname{Eqn}(1.10) P-2$

$$
\emptyset_{s}=\frac{1}{2} \tan ^{-1}\left[\frac{\sigma_{x}-\sigma_{y}}{2 \tau_{x y}}\right]
$$

## What is stress tensor? Explain with neat diagram triaxial stress system and stress, strain relationship for this system.

## Triaxial Stress

When a cubical element of a deformable body is under the action of external forces, a stress would act on each of its six faces. If these stresses are resolved into normal and tangential components to each of the faces, in all nine stresses will act on the element and it is said to be a triaxial stress element. A triaxial stress clement, as shown in Figure 1.12.10, consists of three normal stresses $\sigma_{x}, \sigma_{y}$, and $\sigma_{z}$ and six shear stresses $\tau_{x y}, \tau_{y x}, \tau_{y z}, \tau_{z y}, \tau_{z x}, \tau_{x z}$, all positive. If the element is in equilibrium, then $\tau_{x y}=\tau_{y x}, \tau_{y z}=\tau_{z y}, \tau_{z x}=\tau_{x z}$.


Fig. 1.12.10. A Triaxial Stress Element.


General 3-Dimensional State of Stress

The nine stress components at a point on the element can be represented by a second-order tensor $T$, i.e.

$$
T=\left[\begin{array}{ccc}
\sigma_{x} & \tau_{x y} & \tau_{x z} \\
\tau_{y x} & \sigma_{y} & \tau_{y z} \\
\tau_{z x} & \tau_{z y} & \sigma_{z}
\end{array}\right]
$$

The above matrix is called a stress tensor.

## STRESS CONCENTRATION

## THEORY QUESTIONS:

1. What is stress concentration? How to reduce it?
2. Writes a note on stress concentration factor and methods to reduce stress concentration.
(June/July 2011) (Dec.09/Jan.10) (Marks- 08)
3. What is stress concentration factor? Explain the factors affecting the stress concentration factor?
(Dec.16Jan.17) (Marks- 06)
4. What is stress concentration factor? What are the methods to determine stress concentration factor?
(June/July 2016) (Marks- 06)
5. Give any three examples of stress raisers and show how the stress concentration can be reduced in these cases.
6. explain briefly the following terms:
a. Stress concentration factor
b. Notch sensitivity
c. Fatigue stress concentration factor
(June/July 2009) (Marks- 06)
7. Define stress concentration. Discuss the seriousness of stress concentration in static loading.
(May/June 2010) (Marks- 04)

## NUMERICAL PROBLEMS:

1. Determine the critical stress in a machine component shown in the Fig.
(I P ) (Marks- 10) (May/June 2010)

2. A flat plate subjected to a tensile force of 5 kN is an shown in Fig. the plate material is gray cast iron (good) and the factor of safety is 2.5 . Determine the thickness of the plate. All dimensions are in mm.
(Dec. 15/ Jan. 16)

3. Determine the maximum stress induced in the following cases taking stress concentration into account : A rectangular plate 50 mm wide 8 mm thick and with a central hole of 10 mm is loaded in axial tension of 14.7 kN
(June /July 2013) (04 Marks)
4. A flat plate subjected to a tensile of 5 kN is shown in Fig. the plate material is grey cast iron having $\sigma_{U}$ value of 200 MPa . Determine the thickness of plate. Factor of safety is 2.5. (June /July 2014) (May/June 2010) (08 Marks)

5. What diameter of maximum hole that can be derived in a flat plate shown in Fig. if the stress concentration at step is same as that of a hole.


## (June\July 2013)(06 M)

6. Determine the safe load that can be carried by a bar of rectangular cross section shown in Fig. limiting the maximum stress to 130 MPa taking stress concentration into account.
(Marks- 06) (Dec.13/Jan.14)

7. Determine the maximum stress induced in the following cases taking stress concentration into account:
i. A rectangular plate 50 mm wide, 8 mm thick and with a central hole of 10 mm is loaded in axial tension of 14.5 kN .
ii. A steeped shaft, steeped down from 45 mm to 30 mm with a fillet radius of 6 mm is subjected to a twisting moment of $98 \mathrm{~N}-\mathrm{m}$.
8. A round stepped shaft is made of brittle material cast iron $F G 260$ and subjected to a bending moment of $15 \mathrm{~N}-\mathrm{m}$. As shown in fig. the stress concentration factor at the fillet is 1.5 . Determine the following i). Step diameter ii). Magnitude of stress at fillet iii). Factor of safety.
(June 2012) (Marks- 10)
9. Determine the maximum stress induced in the following cases taking stress concentration into account: A stepped shaft, stepped down from 45 mm to 30 mm with a fillet radius of 6 mm is subjected to a twisting moment of $98 \mathrm{~N}-\mathrm{m}$.
(June /July 2013) (04 Marks)
10. A steeped shaft of diameter ratio 1.2 has a fillet radius of $1 \backslash 10$ of the smaller diameter. It is required to transmit 60 kW at 1200 rpm . Find the suitable diameter of the shaft taking allowable shear stress as 60 MPa .
(Marks- 10) (June/July 2014)
11. A shaft of 50 mm diameter is steeped down to 40 mm with a fillet radius of 5 mm . If the allowable shear stress is $50 \mathrm{~N} / \mathrm{mm}^{2}$, determine the power that can be transmitted at 1200 rpm .
(Marks- 08) (Dec 2010)
12. A cantilever beam is loaded as shown in the Fig. determine the value of ' $d$ ' limiting the maximum normal stress induced to 120 MPa . Analyze at the change of cross section only.
(Dec.13/ Jan. 14) (08 Marks)

13. A shaft made of carbon steel $C 30$ is subjected to the bending moment $2 k N-m$ and has a semicircular groove of radius $0.1 d$. The diameter of shaft is $1.2 d$. Determine the value of ' $d$ ' and dia of shaft assuming factor of safety as 3 .
(I P ) (Marks- 08) (June/July 2011)
14. A steeped shaft is steeped down form 80 mm diameter to 40 mm diameter with a fillet radius of 6 mm . determine the max. stress induced in the shaft by taking stress concentration factor into account if the shaft is subjected to
i) Bending moment of $200 \mathrm{~N}-\mathrm{m}$
ii) Twisting moment of $400 \mathrm{~N}-\mathrm{m}$
(Dec.16/ Jan. 17) (7 M)
15. Determine the maximum stress induced in the semi circular grooved shaft shown in Fig, if it is subjected to
i. An axial load of 40 kN
ii. A bending moment of 400 Nm
iii. A twisting moment of 500 Nm .
(June\July 2013)(7 M)

16. A steeped shaft with its diameter reduced from ' 1.5 d ' to ' d ' d has a fillet radius of 0.1 d . determine the diameter of the shaft and the radius of the fillet to transmit a power of 65 W at a rated speed of 1440 rpm limiting the max shear stress induced to 60 MPa .
(May 2017)
17. A steeped shaft of diameters ratio 1.2 has a fillet radius of $1 / 10$ of the smaller diameter. It is required to transmit 60 kW at 1200 rpm . Find the suitable diameter of the shaft taking allowable shear stress as 60 MPa .
(June-July 2014)
18. A steeped shaft of circular cross section is made of $20 M n_{2}$ steel $\left(\sigma_{y}=\right.$ 432 MPa ). the shaft is steeped down from diameter 1.2 d to d . It is provided with a fillet radius of $0.1 d$ at the change of cross section. The fillet section is at a distance of 350 mm from the fixed end of the shaft. Transverse load of 30 kN is applied at the free end of the shaft the length of the shaft is 600 mm . Determine the value of ' $d$ ' and the fillet radius. Assume a factor of safety of 2.5 and stress concentration factor into account.
(IP)(Marks- 08) (Dec.08/Jan.09) (Dec.07/Jan.08)
19. A steeped shaft is subjected to a transverse load of 8 kN as shown in Fig. the shaft is made of steel with ultimate tensile strength of 400 MPa . Determine the diameter of the shaft based on the factor of safety of 2.
(M E ) (June-July 2007) (12 Marks)

20. A load of 4000 N on a simply supported shaft as in the Fig. find the radius fillet at left side of the shaft, if maximum stress at left fillet is same as that of right. Take $q=0.95$.
(June - July 2016 )(Marks 14)

21. A machine is loaded as shown in Fig. recommended a suitable size for the component, if the allowable stress in bending is limiting to $100 \mathrm{~N} / \mathrm{mm}^{2}$. Take thickness of plate ' t ' $=10 \mathrm{~mm}$.

(June - July 2009)(Marks 14)
22. A tension bar shown in Fig (a), supports an axial load ' $P$ '. it is necessary to replace this member having a hole as shown in the second member. Determine the thickness ' $h$ ' and fillet radius ' $r$ ' at the member by one having a 15 mm hole as shown in Fig (b), so that, the maximum stress will not exceed that of the first member.
(Dec. 2011)(08 M)

23. An infinite plate with an elliptical cutout having major axis 50 mm and minor axis of 25 mm , is subjected to tensile load ${ }^{\prime} F^{\prime}$. Determine the stress concentration factor when
i) The load is perpendicular to major axis.
ii) The load is parallel to the major axis.
(M E )(Dec. 08/ Jan. 09)(08 Marks)

## STRESS CONCENTRATION FACTOR:

INTRODUCTION
Figure 1 shows the stress distribution obtained using finite element technique of a component with discontinuities. If is observed that the stress near the discontinuity (edge, radius and fillet etc.) Is much higher than the stress in the uniform section. The localization of stress near the discontinuity is called stress concentration. The geometric changes (discontinuities) in a member causing the stress concentration are called stress raisers. The region in which these occur is called the area of stressconcentration. Internal cracks and flaws, cavities in welds, blowholes, and pressure at certain points are other common examples of stress raisers.


4.7.

Fig. 1. Str


Some typical illustrations leading to stress concentration.

Similar situation can be visualized for other stress-raisers such as notches, grooves, keyways, etc. The increase in stress depends on the type of loading, the type of material, and the size and shape of the discontinuity.
The designer has to have knowledge of stress concentrations, their causes, technique of preventing them and minimizing serious effects.
Causes:

- Variation in properties.
- Air holes in the metal Impure inclusions
- Blow holes in welds

Flaws and cracks in casting

- Pressure raisers.
- Contact between rails and wheels
- Contact between surface in relative motion like bearing balls and races
- Holding points of the beams by supports
- Rolling contact between gear teeth
- Abrupt change in cross-section
- Scratches due to improper machining
- Damages in improper handling
- Transverse hole in designed member
- Small notches and grooves Small fillet in section changes

Due to the irregularities or due to change in cross section. The stress induced in the member is much higher than the one that is calculated using the known theories.
*** What is stress concentration?
*** Stress concentration is defined as the concentration of stress in the machine member either due to change in cross section or due to irregularities in the member. It is not practically possible to have a machine element without change in cross section. Some examples are key ways in shafts, steps in shaft to accommodate bearings, gears etc.

The stress-concentration factor is used to relate the actual maximum stress to the nominal stress at the discontinuity. It is a ratio of the maximum stress to the nominal stress at the discontinuity.
(i). For tension, stress concentration factor

$$
K_{t}=\frac{\text { maximum stress }}{\text { nominal stress at discontinuity }}=\frac{\sigma_{\max }}{\sigma_{\text {nom }}}
$$

where

$$
\sigma_{\text {nom }}=\frac{F}{A}=\frac{\text { Load }}{\text { Minimum area of cross section }}
$$

(ii). For bending, stress concentration factor

$$
\begin{gathered}
K_{t}=\frac{\text { maximum stress }}{\text { nominal stress at discontinuity }}=\frac{\sigma_{\max }}{\sigma_{n o m}} \\
\sigma_{n o m}=\frac{M \cdot c}{I}=\frac{M}{Z}=\frac{32 M}{\pi \times d^{3}}
\end{gathered}
$$

for a steeped shaft of minimum diameter 'd'subjected to bending
(iii). For torsion, stress concentration factor

$$
\begin{gathered}
K_{s}=\frac{\text { maximum shear stress }}{\text { nominal shear stress at discontinuity }}=\frac{\tau_{\max }}{\tau_{\text {nom }}} \\
\tau_{n o m}=\frac{T \cdot c}{J}=\frac{T}{Z}=\frac{16 T}{\pi \times d^{3}}
\end{gathered}
$$

for a steeped shaft of minimum diameter 'd'subjected to torsion Where, $\quad K_{t}=K_{\sigma}=$ is used for axial and bending stresses.

$$
K_{s}=K_{\tau}=\text { is used for shear (torsional)stresses. }
$$

$\sigma_{\max }$ or $\tau_{\max }=$ Max or Permissible or allowable or working stress.

The effect of a stress raiser on three factors namely

1. Type of load.
2. Material properties.
3. Size and type of discontinuity.

## DESIGN TO REDUCE STRESS CONCENTRATION

Also from the graphs we can draw following general conclusions.

- Stress concentration reduces with increase in fillet radius
- Stress concentration reduces with decrease in step ration.

From these two conclusions we can say that stress concentration can be reduced if the following guidelines are followed.

- Avoid sharp corners by forming them curved shape
- Keep the magnitude of step as low as possible.
- Remove the undesirable material
- Increase the region of stress concentration

There is no expression or formula of force flow analogy to improve the design. It is a method by which designer can qualitatively improve the design of component to safeguard against the failure due to stress concentration.
*** Give any three examples of stress raisers and show how the stress concentration can be reduced in these cases.

## Methods for reducing stress concentration.

1. Providing circular, spiral or parabolic fillet in the abrupt changes in section.
2. Replacing notch by a groove.
3. Cold working increases the strength.
4. Shot penning increases the strength.
5. Proper heat treatment results in increase of life.
6. Grinding of the surface reduces the scratch.
7. Selecting high quality material for the part.
8. Pre- stressing the machine member increases the strength limit.
9. Adding extra drilled holes.
10. Minimizing the material by removing extra material and providing additional notches and grooves.

Methods for reducing stress concentration.

*** What are the methods to determine stress concentration factor? Determination of stress concentration factors

The following are some of the methods used to employ determine the stress concentration factors.

1. Photo elasticity
2. Finite element technique: ( Finite element method)
3. Grid method
4. Brittle coating technique
5. Brittle model
6. Strain gauge method

## *** Discuss the statement "In static loading, stress concentration in ductile materials is not so serious as in brittle material".

ANS: "In static loading, stress concentration in ductile material is not so serious because yielding (plastic deformation) will relieve the stress concentration. Stress concentration in static loading is very serious in brittle materials since there is no yielding (no plastic deformation) or negligible yielding".

## NOTCH SENSITIVITY (q)

The actual stress concentration factor $\left(\mathrm{K}_{\mathrm{t}_{\mathrm{f}}}\right)$ and the Theoretical stress concentration factor $\left(\mathrm{K}_{\mathrm{t}}\right)$ are connected by the relation, by a ratio, known as 'notch sensitivity', q .
$q=\frac{\mathrm{K}_{\mathrm{t}_{\mathrm{f}}}-1}{\mathrm{~K}_{\mathrm{t}}-1} \quad$ Equ (2.4) page -15
$\mathrm{K}_{\mathrm{t}_{\mathrm{f}}}=1+q\left(\mathrm{~K}_{\mathrm{t}}-1\right) \quad$ Equ (2.5) page - 15
where $\quad q=$ Notch sensitivity or index sensitivity.
$\mathrm{K}_{\mathrm{t}}=$ Theoretical stress concentration factor (Form stress factor)
$\mathrm{K}_{\mathrm{t}_{\mathrm{f}}}=$ Actual stress concentration factor
The magnitude of ' $q$ ' depends upon the material and type of loading. It is less for ductile materials and more for brittle materials.

| Material | Notch sensitivity index, $\boldsymbol{q}$ |
| :--- | :--- |
| Alloy steels | $0.85-1.0$ |
| Carbon and low alloy steels | $0.6-0.8$ |
| Aluminium alloys | $0.6-0.75$ |
| Grey cast - iron | $0.1-0.2$ |

## Elliptical hole

There are different methods to find the value of the stress concentration factor for different cases. Theory of elasticity can he used to find out the stress concentration in some cases. The value of $K_{t}$, as determined by theory of elasticity for the case of elliptical hole in a semi-infinite plate ( $\mathrm{w} \gg 2 \mathrm{~b}$ ) subjected to axial load (Below Figure), is given by

$$
K_{t}=1+2 \frac{b}{c} \text { and } \quad K_{t}=1+2 \frac{c}{b} \quad \text { Table } 2.19
$$

Where, $\quad b=$ semi major axis,$\quad c=$ semi minor axis

## COMBINED STRESS:

## NUMERICAL PROBLEMS:

1. A round steel rod is subjected to a tensile load of $90 k N$. Taking the yield stress for the steel as 328.6 MPa and factor of safety as 1.8 , determine suitable diameter for the rod.
2. A machine shaft member of 60 mm diameter, 300 mm long and supported at one end subjected to a tensile load of 50 kN . Find the tensile and shear stresses induced in the member.
3. Determine the Max. Stress induced in a link loaded as shown in Fig. 3.

4. Figure 4 shows a machine link made of steel ( $\sigma_{y}=234 \mathrm{MPa}$ ) subjected to a tensile load of 45 kN . Find the required thickness of the link taking factor of safety
as 3 .


Fig. 4
5. A beam of uniform rectangular cross section is fixed at one end and carries a transverse load of 1.8 kN at a distance of 0.9 m from the fixed end. The material used is $C 30$ steel ( $\sigma_{\mathrm{y}}=294 \mathrm{MPa}$ ) and factor of safety is 2.5 . Find the width and depth of $\mathrm{c} / \mathrm{s}$ if the depth is twice the width.
6. A 1 mm thick steel blade is bent into a circular arc of 500 mm radius. Determine the bending stress induced mid the bending moment required to bend the blade of width 15 mm . Take $E=210 \mathrm{GPa}$.
7. A machine element on the form of a cantilever beam has a rectangular cross section of depth 200 mm . this beam is subjected to an axial tensile load of 60 kN and a transverse load of 50 kN acting downwards at the free end of the beam, which has span of 800 mm . determine the width of the rectangular cross section, if the machine element if made up of steel with allowable tensile stress of 80 MPa .
(May 2017)(08 Marks)
8. A machine shaft is subjected to a bending moment of $3 \mathrm{kN}-\mathrm{m}$ and a torque of 1.5 $\mathrm{kN}-\mathrm{m}$. find the suitable diameter of the shaft if the allowable normal and shear stresses for the material used are 120 MPa and 75 MPa respectively.

## (June/July 2014)(08 Marks)

9. Find the diameter ' $d$ ' and the dimensions ' $b$ ' and ' $t$ ' of the machine member shown in Fig. the permissible normal stress in the member is 80 MPa and the permissible shear stress is 55 MPa . Assume $\mathrm{b} / \mathrm{t}=5$.

(Dec.09/Jan.10)(10 M)
10. A bolt in an assembly is subjected to a pull of 1000 N along its axis and a shear force of 500 N , what will be the maximum stress induced in the bolt. If the bolt is made of SAE 1045 annealed steel, is the bolt is safe given that the diameter of bolt is 12 mm .
(June/July 2009)(7 M)
11. A 50 mm diameter steel rod supports a 9000 N load and in addition is subjected to torsional moment of 100 N.m. determine the maximum normal and the maximum shear stress.
(June/July 2009)(8 M)
12. Determine the stress induced for the loading member as shown in Fig. 7.

13. Design a spindle of milling machine to transmit 15 kW at 1000 rpm . The angular twist is not to exceed $0.5^{0}$ per meter length. The material for the spindle is $C 45$ steel ( $\sigma_{\mathrm{y}}=353$ ). the outside diameter of the spindle is twist that of inside diameter. Take factor of safety $=2$.
14. A circular shaft 50 mm diameter fixed at one end is subjected to an axial load of 20 kN and a torque of $1.5 \mathrm{kN}-\mathrm{m}$. If the length of the shaft is 300 mm , determine the nature and magnitude of stresses at the critical point.
(Dec. 06 / Jan. 07)(10 Marks)
15. A 50 mm diameter steel rod supports a 9000 N load and in addition is subjected to torsional moment of $100 \mathrm{~N}-\mathrm{m}$. determine the maximum normal and the maximum shear stress
(June/July 2009)(08 Marks)
16. A circular rod of diameter 60 mm and length 200 mm is fixed at one end. The free end is subjected to an axial tensile load of 10 kN , a transverse load of 6 kN and a twisting moment of $400 N-m$. Determine the stresses at the critical points.
(Dec 2011)(10 Marks)
17. A hollow shaft of 40 mm diameter and 25 mm inner diameter is subjected to a twisting moment of $118 \mathrm{~N}-\mathrm{m}$, a axial thrust of 9806 N and a bending moment of $79 \mathrm{~N}-\mathrm{m}$. Calculate the maximum compressive and shear stresses.
(June/July 2013)(10 Marks)
18. A cantilever circular rod of diameter 50 mm and length 300 mm . find out the values of principle stress and maximum shear stress under the following conditions:
i. Applying an axial load of 20 kN .
ii. Applying 4 kN load at end, acting downwards creating bending stresses.
iii. Applying a torque of $1.5 \mathrm{kN}-\mathrm{m}$.

## COMBINED STRESS

## DEFINITION

A part subjected to two types of loading simultaneously gets a combination of stresses induced at a point in the cross section. The design equations are based on the failure of the component at that point where these combined stresses exceed the limit.

## COMBINED BENDING MOMENT AND AXIAL LOAD

A hypothetical member subjected to two loads simultaneously may be treated as a member subjected to $P_{1}$ and $P_{2}$ separately. The stresses under $P_{1}$ and $P_{2}$, are calculated separately and superimposed on each (Fig. 2.3 and Fig. 2.4.)


$$
\begin{equation*}
\sigma_{b}= \pm \frac{P_{2} . l . c}{I} \quad \sigma_{t}=\frac{P_{1}}{A} \tag{1.1}
\end{equation*}
$$


$\therefore$ Total induced stress at the top fiber

$$
\sigma=\sigma_{t}+\sigma_{b}=\frac{P_{1}}{A}+\frac{M}{Z}=\frac{P_{1}}{A}+\frac{P_{2} \cdot l}{Z}
$$

and
Total induced stress at the bottom fiber
$\sigma=\sigma_{t}-\sigma_{b}=\frac{P_{1}}{A}-\frac{M}{Z}$

$$
\sigma=\frac{P_{1}}{A}-\frac{P_{2} \cdot l}{Z}
$$

Eqn (1.17) P-3

## Design equation

Design equation for the component may be written as

$$
\frac{\sigma_{y}}{N}=\frac{P_{1}}{A} \pm \frac{M}{Z}=\frac{P_{1}}{A} \pm \frac{P_{2} \cdot e}{Z}
$$

When stresses due to bending and direct loading both are tensile.

## ECCENTRIC LOADING: NUMERICAL PROBLEMS:

1. A short cylindrical hollow tube having an external diameter of 80 mm and an internal diameter of 60 mm is subjected to a longitudinal compressive force of 120 kN . The force acts in a line parallel to the axis of the tube. Determine the minimum stress in the section and the maximum eccentricity of the load, if the maximum compressive stress is not to exceed 90 MPa .
2. A 50 mm diameter steel rod supports a 9.0 KN load and in addition is subjected to a torsional moment of $100 \mathrm{~N}-\mathrm{m}$ as shown in Fig 2.11. Determine the maximum tensile stress and the maximum shear stress.
(June/July 2014) (Dec.14/Jan 2015) (09 Marks)

3. Determine the required thickness of the steel bracket at section $A-A$. When loaded as shown in fig 2.12 in order to limit the tensile stress to 100 MPa .
(June 2012)(06 Marks)
4. Determine the maximum stress induced in the wall bracket shown in Fig. 16

5. Determine the thickness ' $b$ ' of the bracket at section A-A, when loaded as shown in Fig. in order to limit the maximum tensile stress to 70 MPa .

(June 2012)(10 M)
6. A steel bracket as shown in Fig. 17. is loaded with two 25 kN forces. The weights of the bracket and stress concentration are to be neglected. If the maximum tensile stress is not to exceed35 MPa, what is the minimum value of the dimension ' $h$ '.


Fig. 17
7. The load ' $F$ ' on C clamp shown in 2.15 is 37.5 kN . The clamp is made of cast steel with $b=3 h$ and $e=$ 200 mm . If the allowable stress is 100 MPa . Determine the dimensions $b \& h$.


Fig. 18
8. A steel member is loaded as shown in Fig 19. Determine the magnitude of (i). Maximum normal stress (ii) Minimum normal stress (iii) Maximum shear stress
(July 2007)(12 Marks)

9. A bracket shown in Fig 20 is subjected to a pull of 15 kN at $60^{\circ}$ to the vertical. Determine the maximum tensile stress in the bracket.
(Dec 2010)(12 Marks)

10. A mild steel bracket shown in fig 21 is subjected to a pull of 10 kN . The bracket has a rectangular cross - section whose depth is twice the width. If the allowable stress for the material is $80 M P a$, determine the cross section of the bracket.

11. A wall bracket of rectangular cross - section whose depth is twice of width carries a load of 60 kN as shown in Fig. (21.1). find the required width and depth of cross section taking allowable stress as 90 MPa .
(Dec.15/Jan.16) (June/July 2014)(10 Marks)

12. Determine the dimensions of the section for the bracket as shown in Fig. 22. The permissible stress of the material is $65 \mathrm{~N} /$ $\mathrm{mm}^{2}$.

13. A C-clamp carries a load of 20 kN . The frame is as shown in Fig. P.23. and is of uniform section at all points, i.e., $150 \mathrm{~mm} \times 25 \mathrm{~mm}$ thickness. Find the stresses at Section $Y-Y$, Section $X-X$ and Section $S-S$.

14. Determine normal stresses at the extreme fibers on the cross section $A-A$ of $C$ - clamp loaded as shown in 24.
(July 2006)(14 Marks)

15. Determine dimensions principle cross - section of the cast iron link shown in fig 25. The maximum tensile load is $37 k N$. The stress values should not be exceeded $21 M P a$ in tension and $84 M P a$ in compression.

16. Find the numerical maximum normal
stress and the maximum shear stress at section $x-x$ in the member loaded as shown in Fig. 26.

17. Determine the maximum normal stress and maximum shear stress at section $A-A$ of the crank shaft shown in Fig. P.27. when a load of 10 kN is assumed to be concentrated at the center of crack pin. Neglect the effect of transverse shear.
(June/July 2008)(08 Marks)

18. An overhang crank with pin and shaft is as shown in Fig. A tangential load of 15 kN acts on the crack pin. Determine the diameter at section ' XX ' using maximum shear stress theory. The crack is made of C60 carbon steel. Take factor of safety as 2. All dimensions are in mm. (Dec.15/Jan.16)(05 Makes)

19. Determine the maximum normal stress and maximum shear stress at section $A-A$ of the crank shaft shown in Fig. P.28. neglect the effect of transverse shear.


## ECCENTRIC LOADING

Consider a loading as shown in Fig. 2.2. Here, the load axis does not coincide with the geometrical axis of the member. Such a case of loading is referred to as a case of eccentric loading,


Eccentric Loading
The analysis for such situations can be done by the following method: (Refer Fig.)


Consider an equal and opposite imaginary force acting passing through the centroidal axis. [Refer Fig. (a)]

The load situation can now be considered equivalent to:
a). A Direct load ' F ' acting through the centroidal axis and
b). A Couple of magnitude ' $F \times e$ ', which will cause bending as shown in Fig.. (b).

Thus an eccentrically loaded member will be analyzed as a combination of direct load and bending. In the present case, the direct load creates compressive stresses.
$\sigma_{c}=-\frac{F}{A}$
Eqn
(1.1) $P-2$
[Fig. (c)]

Due to bending moment, $M=F$. $e$, the bending stress will be :
$\sigma_{b}= \pm \frac{\text { F.e.c }}{I} \quad \operatorname{Eqn}$ (1.16) P-3 [Fig. (d)]
The combined stress

$$
\begin{equation*}
\sigma=-\frac{F}{A} \pm \frac{M}{Z}=-\frac{F}{A} \pm \frac{F \cdot e}{Z} \tag{Eqn}
\end{equation*}
$$

Thus the combined stress distribution will be: $\sigma=-\frac{F}{A}+\frac{M}{Z}=-\frac{F}{A}+\frac{F . e}{Z}$ [Fig. 2. 2. (e), at point A]
and $\quad \sigma=-\frac{F}{A}-\frac{M}{Z}=-\frac{F}{A}-\frac{F . e}{Z}$ [Fig. 2. 2. (e), at point B]
Thus the maximum stress in the present case will be compressive and would occur at point B.

A frame of C clamp as shown in Fig. Fig-2.5. is also subjected to direct tensile stress as well as bending stress and the equations to be used are
$\frac{\sigma_{y}}{N}=\frac{F}{A}+\frac{M}{Z}=\frac{F}{A}+\frac{F . e}{Z}$
$\frac{\sigma_{y}}{N}=\frac{F}{A}-\frac{M}{Z}=\frac{F}{A}-\frac{F . e}{Z} \quad$ For outer fiber and ductile material,


## Module -2

Design for Impact and Fatigue Loads
Impact Loads :Impact stress due to Axial, Bending and Torsional loads.
Fatigue failure: Endurance limit, S-N Diagram, Low cycle fatigue, High cycle fatigue, modifying factors: size effect, surface effect. Stress concentration effects, Notch sensitivity, fluctuating stresses, Goodman and Soderberg's relationship, stresses due to combined loading, cumulative fatigue damage.

## IMPACT LOADING:

## NUMERICAL PROBLEMS:

1. Derivation of instantaneous stress due to axial impact on bars.
(June 2012)(05 Marks)
2. A cantilever beam 12 mm deep, 8 mm wide and 300 mm long is loaded as follows, at the free end. Determine the maximum bending stresses in each case.
i. A load of 50 N applied gradually.
(Dec.2010)(10 M)
ii. A load of 50 N dropped through a distance of 5 mm .
3. Design a rod of solid circular cross section of length 200 mm (placed vertical) to sustain an axial compressive load of 1000 N , that falls on it from a height of 10 mm . the material selected has a design stress of 80 MPa and $E=2.1 \times$ $10^{5} \mathrm{MPa}$.
(Dec.15/Jan.16)(03 Makes)
4. A weight of 20 kN falls from a height of 300 mm on to a vertical steel pole of 6 meter long and 0.3 meter in diameter. The pole is fixed at lower end. Take the modulus of elasticity of steel as 206 GPa. Determine
i. Maximum compressive stress in the pole.
ii. Deformation of the pole due to impact load.
iii. Energy absorbed by the pole. (Dec.08/Jan.09)(12 Makes)
5. A weight of 20 kN falls from a height of 30 mm on to a vertical steel pole of 6 m long having diameter of 0.3 m . The pole is fixed at lower end. Take the modulus of elasticity of steel as 206 GPa . Determine the maximum instantaneous stress produced and maximum instantaneous deflection.
(June/July 2014) (Marks
10) 
6. A steel rod of 1.5 meters long resists on impact load of 2 kN dropped through a distance of 50 mm along its axis, limiting the maximum stress in the rod to 150 MPa, determine
i. The diameter of rod required
ii. Impact factor. Use $E=200$ GPa. (June/July 2009) (Marks 10)
7. The brasses of an automobile engine connecting rod have worn, so as to allow play which gives shock loading equivalent to a weight of 5886 N falling through a height of 0.2 mm . The connecting rod is 250 mm long and has a cross sectional area of $3 \times 10^{-4} \mathrm{~m}^{2}$. determine the stress induced in the connecting rod. Compare the maximum stress induced with that of a static load of 5886 N .
(June/July 2013)(06 Marks)
8. An unknown weight fall through 10 mm on a collar rigidly attached to the lower end of a bar 3 m long and $600 \mathrm{~mm}^{2}$ in section. If the maximum instantaneous extension is 2 mm , what is the corresponding stress and the value of unknown weight? TakeE $=206$ GPa.
(Dec. 2011) (June/July 2011) (Marks 08) (Marks 07)
9. An unknown weight fall through 15 mm on a collar rigidly attached to the lower end of a vertical bar 2 m long and 500 sq . mm. If the maximum instantaneous extension is 2 mm , what is the corresponding stress and the value of unknown weight? Take $E=200$ GPa. (Dec.16/Jan.17)(8 Marks)
10. A weight of 1.5 kN falls through a height of 6 mm before it strikes a collar provided at the lower end of a vertical steel rod as shown in Fig. the diameter and the length of rod are 50 mm and 0.6 m respectively. The modulus of elasticity of steel is 206 GPa. Neglecting the inertia effect of the masses, determine,
i. Stress induced in the rod by neglecting the inertia effect of the rod.
ii. Stress induced in the rod when the same load acts statically.
iii. How much the rod length can be changed before the impact stress exceeds 120 MPa .
iv. How much can the height of fall be increased before the impact stress exceeds 120 MPa by retaining its original length.
v. Stress induced in the rod by considering the inertia of the rod. Take the specific weight of the material of the $\operatorname{rod}$ as $76.6 \mathrm{kN} / \mathrm{m}^{3}$.
11. A steel cantilever beam of rectangular cross section is loaded 400 mm from the support. The width of the beam is 15 mm and depth is 20 mm . Determine the maximum bending stress in the beam, when a weight of 100 N is dropped on the beam through a height of 5 mm . Take $E=10 \mathrm{GPa}$.
(Dec. 2010)(Marks 06)
12. A machine member can be considered as simply supported beam of 1 m length. Cross section of the beam is $60 \mathrm{~mm} \times 60 \mathrm{~mm}$ square. Determine the instantaneous maximum defection and bending stress if a mass of 15 kg falls from a height of 250 mm at the mid point of the beam made of steel.
(Dec. 2012)(05 M)
13. A steel cantilever beam of rectangular cross section is loaded 400 mm from the support. The width of the beam is 15 mm and depth is 20 mm . Determine the maximum bending stress and the deflection in the beam, when a weight of 100 N is dropped on the beam through a height of 5 mm . take $E=$ 10 GPa . Also find above values by considering the effect of inertia.
14. A machine element in the form of a cantilever beam has a rectangular cross section of 40 mm width and 120 mm depth. The span of the beam is 600 mm . A transverse load of 5 kN falls from a height of ' $h$ ' at the free end of the beam. Determine safe value of ' $h$ ' limiting the maximum normal stress induced in the machine element, due to impact, to 120 MPa . The modulus of elasticity of the material of the beam is 210 GPa. (July 2006)(Marks 10)
15. Determine the cross section dimension for the cantilever of length 200 mm , when it is subjected to impact by a weight of 1000 N falling through 1 mm height at the free end. Take depth of section as 4 times the width and allowable stress of $80 \mathrm{~N} / \mathrm{mm}^{2} \quad \quad($ June $/$ July 2009) (Marks 09)
16. A weight 600 N drops through a height of 20 mm and impacts the center of 300 mm long simply supported circular cross section beam. Find the diameter of the beam another maximum deflection, if the allowable stress is limited to 90 MPa . Neglect the inertia effect and take $E=200 \mathrm{GPa}$.
17. A 5 kg block is dropped from a height of 200 mm on to a beam as shown in Fig. the material has an allowable stress of 50 MPa . Determine the dimensions of the rectangular $\mathrm{C} / \mathrm{S}$ whose depth is 1.5 times the width. Take $E=70 \mathrm{GPa}$.
(Dec.07/Jan.08)(10 Marks)
18. A beam of 400 mm depth I -section is resting on two supports 6 m apart. It is loaded by a weight of 5000 N falling through a height of 10 mm and striking the beam at mid point. Moment of inertia of the section is $12 \times$ $10^{7} \mathrm{~mm}^{4}$. Take $E=2 \times 10^{5} \mathrm{MPa}$. Determine,
i) Impact factor
ii) Instantaneous Max. stress
iii) Instantaneous Max. deflection
iv) Instantaneous Max. load
(May 2017)(8 M)
19. A steel rod 1.5 m long has to resist longitudinally an impact of 2.5 kN falling under a gravity at a velocity of $0.9925 \mathrm{~m} / \mathrm{s}$. the maximum computed stress is to be limited to 150 MPa . Determine the diameter of the round rod.
(June July 2013)(10 M)
20. A solid flywheel of 120 mm diameter and 20 mm thick is mounted on a 20 mm diameter overhanging steel shaft. The length of the shaft from the nearest bearing is 250 mm . The flywheel runs at 2400 rpm and the density of the flywheel material is $2000 \mathrm{~kg} / \mathrm{m}^{3}$. Estimate the resulting maximum torsional stress and deflection in the shaft if it is desired to stop the flywheel instantly. Take the modulus of rigidity of steel shaft as 79 GPa. Neglect the weight of the shaft.
21. A mass of 500 kg is being lowered by means of steel wire rope having cross sectional area of $250 \mathrm{~mm}^{2}$. The velocity of the weight is $0.5 \mathrm{~m} / \mathrm{sec}$. When the length of the extended rope is 20 m , sheave gets stuck up. Determine the stresses induced in the rope due to sudden stoppage of the sheave. Neglect friction. Take $E=190 G P a$. (Dec.14/Jan 2015) (July 2007) (Marks 08)
22. A weight of $2 k N$ is being lowered with a velocity of $2 \mathrm{~m} / \mathrm{sec}$ with the help of a wire rope and a sheave. When the sheave stops suddenly after the weight has reached a distance of 10 m , find the maximum stress in the rope. The area resisting the stress is $636 \mathrm{~mm}^{2}$ and modulus of elasticity is 190 GPa . Neglect inertial effect.
(Dec.06/jan.07)(10 Makes)
23. An elevator car carrying a load of 10 kN is descending by means of a steel rope at a speed of $1 \mathrm{~m} / \mathrm{sec}$. The cross section area of the rope is $400 \mathrm{~mm}^{2}$. The rope is suddenly brought to rest by braking after 30 seconds of decent. Calculate the stress induced in the rope due to rope due to sudden stoppage, if the young's modulus for the rope is 80000 MPa
(July/Aug. 2004)
24. Determine the maximum torsional impact that can withstand, without permanent deformation by a 100 mm cylindrical shaft 5 m long and made of SAE 1045 annealed steel $\left(\tau_{y}=180\right.$ MPa and $\left.G=82 G P a\right)$. Factor of safety $=3$.
(May/June 2010) (Marks 04)
25. Determine the impact shear stress and the angular deflection of the machine element of diameter 20 mm shown in Fig. when 1 kN weight is dropped at a height of 10 mm . Take $G=80 \mathrm{GPa}$.
26. Determine the maximum torsional impact that can withstand, permanent deformation by a 100 mm cylindrical shaft 5 m long and made of SAE 1045 annealed steel $\left(\tau_{y}=180 \mathrm{MPa}\right.$ and $\left.G=82 \mathrm{GPa}\right)$. Factor of safety $=3$.
(May/June 2010)(08 M)

## IMPACT LOADING:

## Introduction:

A static load is slowly applied, gradually increasing from zero to its maximum value; therefore, the load remains constant.

If a moving body strikes another body, the second body is subjected to an impact which is equal to the kinematic energy of moving body. The stress induced due to impact load is called impact stress. Examples of machines that are subjected to impact loads are forging machines; presses are used for blanking operation, presses used for impact extrusion.

## Impact energy:

If the weight of moving body is ' $W$ ' and its velocity is ' $v$ ' then

$$
\text { kinetic energy, } \mathrm{E}_{\mathrm{k}}=\frac{W v^{2}}{2 g} \quad \boldsymbol{E}-\mathbf{2 . 3 6}(\mathbf{2 0} \mathrm{A})
$$

Impact energy of the body $\mathrm{E}_{\mathrm{k}}=W h \quad \boldsymbol{E}-\mathbf{2 . 3 7 ( 2 0 A )}$

For stress calculation it is convenient to express the impact energy as if it was produced by a falling body. If ' $h$ ' is the height of fall and ' $v$ ' is the velocity of fall, then

Compare above two equation $\mathrm{h}=\frac{v^{2}}{2 g}$
The total work done by the falling body can also be calculated by using equation (i) if ' $h$ ' includes both the height of fall before contact and the deformation of the body.

## Effect of sudden loads:

Let $\quad \mathrm{W}=$ suddenly applied load, $\quad \mathrm{A}=$ area of cross - section. $e^{\prime}=$ Deformation in the member due to the sudden load.
$\mathrm{E}=$ modulus of elasticity. $\quad l=$ length of member.
$\sigma^{\prime}=$ Stress induced in the member due to the sudden load.
The work done due to sudden load $=W \times e^{\prime}$
Strain energy, $U=\frac{1}{2} F e^{\prime}$ where F is the force at which $e^{\prime}$ is produced.
$\therefore U=\frac{1}{2}\left(\sigma^{\prime} A\right) e^{\prime}=\frac{1}{2}\left(\sigma^{\prime} A\right)\left(\sigma^{\prime} \frac{l}{E}\right)=\frac{1}{2} \frac{\sigma^{\prime 2} A l}{E}=\frac{\sigma^{\prime 2} V}{2 E}$
$U=\frac{\sigma^{2}{ }^{2} V}{2 E} \quad \boldsymbol{E}-2.48(P-21)$
Where $V=$ Volume $=V l$
Now equating the strain energy to work done.

$$
\begin{gathered}
\frac{\sigma^{\prime 2} V}{2 E}=W \times e^{\prime} \\
\frac{\sigma^{\prime 2} V}{2 E}=W \times \frac{\sigma^{\prime} l}{E} \quad \therefore d l=\frac{W l}{A E}=\frac{\sigma l}{E} \\
\sigma^{\prime}=\frac{2 W}{A}=2 \sigma
\end{gathered}
$$

Where $\sigma^{\prime}$ is impact stress and $e^{\prime}$ is deformation due to impact load.

$$
\sigma^{\prime}=2 \sigma \quad E-2.45 a(P-20 A)
$$

$\sigma$ is static stress and ' $e$ ' is deformation due to static load.

$$
e^{\prime}=2 e \quad E-2.45 b(P-20 A)
$$

Then the stress and the strain are twice that of those developed by a gradually applied load is other words suddenly applied load is eqvalent to twice the static load.

## Axial impact on bars:

## Derivation of instantaneous stress due to axial impact

The stress produced in the member due to falling load is known as impact stress. Consider a load ' $W$ ' is dropped from a height of ' $h$ ' on the end of a vertical bar as shown as fig. which produces instantaneous deformation $e^{\prime}$ or $\delta_{\max }$ and stress $\sigma^{\prime}$ or $\sigma_{\max }$

Let $\quad e^{\prime}=$ Deformation in the bar due to the impact load.
$\mathrm{E}=$ modulus of elasticity of the material of the bar.
$\mathrm{F}=$ force at which the deformation $e^{\prime}$ is produced.
$\sigma^{\prime}=$ Stress induced in the bar due to the impact load.
$l=$ length of member.
$\mathrm{A}=$ area of cross - section.
$\mathrm{W}=$ weight of falling body.
$\sigma=$ Static Stress due to weight 'W'.
$e=$ Static Deformation due to weight 'W'.


The work done by the falling body is equal to its change in potential energy.

$$
\text { Work done }=W\left(h+e^{\prime}\right) \quad \boldsymbol{E}-2.38 \boldsymbol{a}(\boldsymbol{P}-20 \boldsymbol{A})
$$

Since the energy gained by the system is equal to the potential energy lost the weight

$$
\begin{equation*}
\frac{1}{2} F e^{\prime}=W\left(h+e^{\prime}\right) \quad \text { where } F=\sigma^{\prime} A \quad \text { and } e^{\prime}=\sigma^{\prime} \frac{l}{E} \tag{i}
\end{equation*}
$$

Substituting theses values in equation (i)

$$
\frac{1}{2}\left(\sigma^{\prime} A\right)\left(\sigma^{\prime} \frac{l}{E}\right)=W\left(h+\sigma^{\prime} \frac{l}{E}\right)
$$

i.e., $\frac{A l}{2 E} \sigma^{\prime 2}-\frac{W l}{E} \sigma^{\prime}-W h=0 \quad$ Quadratic equation $\frac{b \pm \sqrt{(b)^{2}+4 a c}}{2 . a}$
$\therefore \sigma^{\prime}=\frac{\frac{W l}{E} \pm \sqrt{\left(\frac{W l}{E}\right)^{2}+4\left(\frac{A l}{2 E}\right) \cdot W h}}{2 \cdot \frac{A l}{2 E}}=\frac{\frac{W l}{E} \pm \frac{W l}{E} \sqrt{1+2 A h \cdot \frac{E}{W l}}}{\frac{A l}{E}}$

$$
\therefore \sigma^{\prime}=\frac{W}{A} \pm \frac{W}{A} \sqrt{1+\frac{2 h E A}{W l}}
$$

Considering positive sign, $\sigma^{\prime}=\frac{W}{A}\left(1+\sqrt{1+\frac{2 h E A}{W l}}\right) \quad \boldsymbol{E}-2.39(\boldsymbol{P}-20 \mathrm{~A})$

$$
\sigma^{\prime}=\sigma\left(1+\sqrt{1+\frac{2 h}{e}}\right) \quad \boldsymbol{E}-2.39(P-20 A) \quad \text { where }\left[d l=e=\frac{W l}{A E}\right]
$$

Now multiplying both sides of the equation $(\boldsymbol{E}-\mathbf{2 . 3 9})$ by $\frac{l}{E}$ the equation becomes $\sigma^{\prime} \frac{l}{E}=\frac{W}{A} \cdot \frac{l}{E}\left(1+\sqrt{1+\frac{2 h E A}{W l}}\right)$
$\therefore \quad e^{\prime}=e\left(1+\sqrt{1+\frac{2 h}{e}}\right) \quad \boldsymbol{E}-2.40(\boldsymbol{P}-20 \boldsymbol{A}) \quad$ where $\left[d l=e=\frac{W l}{A E}\right]$

## Impact factor;

The ratio of maximum stress to the static stress is called impact factor.
For axial loads, impact factor $=\frac{\sigma^{\prime}}{\sigma}=1+\sqrt{1+\frac{2 h}{e}}$

## Impact bending:

## Derivation of instantaneous stress due to impact bending.

Consider a lad ' W ' is dropped from a height of ' $h$ ' on a simple supported beam as shown as fig. which produces instantaneous deflection $y^{\prime}$ or $y_{\max }$ and stress $\sigma_{b}{ }^{\prime}$ or $\sigma_{b_{\text {max }}}$

Let $E=$ modulus of elasticity of the material of the bar
$\sigma^{\prime}=$ Stress induced in the beam due to the impact load.
$y^{\prime}=$ Deflection in the beam due to the impact load.
$\sigma=$ Bending Stress due to Static weight 'W'.
$y=$ Deflection due to Static weight ' $W$ '.
$\mathrm{W}=$ weight of falling body.
$\mathrm{A}=$ area of cross - section.
$\mathrm{I}=$ M.I. of the beam


The work done by the falling body is equal to its change in potential energy.

$$
\text { Work done }=W\left(h+y^{\prime}\right) \quad \boldsymbol{E}-2.38 \boldsymbol{a}(\boldsymbol{P}-20 \boldsymbol{A}) \ldots(\boldsymbol{i})
$$

Let ' $W_{e}$ ' be the equivalent static load which produces the same amount of deflection ' $y$ ', then, $S$ train energy $=\frac{1}{2} W_{e} y^{\prime}$

Equating the two equations

$$
\begin{align*}
& \frac{1}{2} W_{e} y^{\prime}=W\left(h+y^{\prime}\right) \\
& \therefore \quad W_{e}=2 W \frac{\left(h+y^{\prime}\right)}{y^{\prime}} \tag{iiii}
\end{align*}
$$

The deflection of the bam under the static equivalent load $W_{e}$ at the center is given by

$$
\begin{equation*}
y^{\prime}=\frac{W_{e} l^{3}}{48 E I} \tag{iv}
\end{equation*}
$$

Substituting theses value of ${ }^{\prime} W_{e}{ }^{\prime}$ in equation (iv)

$$
\begin{equation*}
y^{\prime}=\frac{2 W\left(h+y^{\prime}\right)}{y^{\prime}} \frac{l^{3}}{48 E I}=\frac{2\left(h+y^{\prime}\right)}{y^{\prime}} \frac{W l^{3}}{48 E I} \tag{v}
\end{equation*}
$$

But, $\frac{W l^{3}}{48 E I}=y=$ static deflection due to weight $W$ Table 1.4 $(\boldsymbol{P}-\mathbf{1 0})$

Equation (v) becomes

$$
y^{\prime}=\frac{2\left(h+y^{\prime}\right)}{y^{\prime}} \cdot y
$$

$$
\begin{gathered}
\text { i.e., } \quad y^{\prime 2}=2 h y+2 y^{\prime} y \quad \text { i.e., } \quad y^{\prime 2}-2 h y+2 y^{\prime} y=0 \\
\therefore y^{\prime}=\frac{2 y \pm \sqrt{(2 y)^{2}+4(1) .2 h y}}{2 \times 1}
\end{gathered}
$$

Taking positive sign

$$
\begin{gathered}
y^{\prime}=\frac{2 y+\sqrt{4 y^{2}+4 \times 2 h y}}{2}=\frac{2 y+2 y \sqrt{1+\frac{2 h}{y}}}{2}=\frac{2 y\left[1+\sqrt{1+\frac{2 h}{y}}\right]}{2} \\
y^{\prime}=y\left[1+\sqrt{1+\frac{2 h}{y}}\right] \quad \boldsymbol{E}-\mathbf{2 . 4 2}(\boldsymbol{P}-\mathbf{2 0} \boldsymbol{A})
\end{gathered}
$$

Multiply both sides of equation $(\boldsymbol{E}-\mathbf{2 . 4 2})$ by $\frac{8 E . c}{l^{2}}$, the equation becomes,

$$
\begin{array}{r}
y^{\prime} \times \frac{8 E . c}{l^{2}}=y \times \frac{8 E . c}{l^{2}}\left[1+\sqrt{1+\frac{2 h}{y}}\right] \\
\text { i.e., } \sigma^{\prime}=\sigma\left(1+\sqrt{1+\frac{2 h}{y}}\right) \quad E-2.41(P-20 \mathrm{~A})
\end{array}
$$

## Impact factor;

The ratio of maximum stress to the static stress is called impact factor.
For bending loads, impact factor

$$
=\frac{\sigma^{\prime}}{\sigma}=1+\sqrt{1+\frac{2 h}{y}}
$$

Note: Deflection at the center of beam $y=\frac{l^{2}}{8 R}$

$$
\begin{gathered}
\text { New, } \frac{M}{I}=\frac{E}{R} \\
\therefore R=\frac{E I}{M} \\
y=\frac{l^{2}}{8}\left(\frac{E I}{M}\right) \\
\text { Also, } \frac{M}{I}=\frac{\sigma_{b}}{c} \\
\therefore \quad y=\frac{l^{2}}{8}\left(\frac{\sigma_{b}}{E . c}\right) \\
\therefore \quad \sigma_{b}=y \times \frac{8 E . c}{l^{2}}
\end{gathered}
$$



## Derivation of instantaneous stress due to impact bending.

Consider a lad ' $W$ ' is dropped from a height of ' $h$ ' on a simple supported beam as shown as fig. which produces instantaneous deflection $y^{\prime}$ or $y_{\max }$ and stress $\sigma_{b}{ }^{\prime}$ or $\sigma_{b_{\text {max }}}$

Let $\quad \mathrm{E}=$ modulus of elasticity of the material of the bar.
$\sigma^{\prime}=$ Stress induced in the beam due to the impact load.
$y^{\prime}=$ Deflection in the beam due to the impact load.
$\sigma=$ Bending Stress due to Static weight 'W'.
$y=$ Deflection due to Static weight 'W'
$\mathrm{W}=$ weight of falling body.
$A=$ area of cross - section
$l=$ length of member.
$\mathrm{I}=$ M.I. of the beam
$\mathrm{M}=\mathrm{B} . \mathrm{M}$. of the beam.


The work done by the falling body is equal to its change in potential energy.

$$
\begin{equation*}
\text { Work done }=W\left(h+y^{\prime}\right) \quad \boldsymbol{E}-\mathbf{2 . 3 8 a} \boldsymbol{a}(\boldsymbol{P}-\mathbf{2 0} \boldsymbol{A}) \tag{i}
\end{equation*}
$$

Static bending stress $\sigma_{S t b}=\frac{M}{Z}=\frac{M . c}{I}=\frac{w l . c}{I}$
Energy absorbed by the beam $U=\frac{\sigma_{b}{ }^{\prime} I}{l c} \times \frac{y^{\prime}}{2}$

Equating the energy absorbed to potential energy, we get

$$
\begin{aligned}
& \frac{\sigma_{b}^{\prime} I}{l c} \times \frac{y^{\prime}}{2}=W\left(h+y^{\prime}\right) \\
& \frac{\sigma_{b}^{\prime} I}{W l c} \times y^{\prime}=2\left(h+y^{\prime}\right)
\end{aligned}
$$

$$
\text { i.e., } \quad \frac{\sigma_{b}^{\prime}}{\sigma_{b}} \times y^{\prime}=2\left(h+y^{\prime}\right) \quad \ldots \ldots .(\text { iv }) \text { becouse }\left(\sigma_{\text {St } b}=\frac{w l . c}{I}\right)
$$

For elastic material, $\quad \frac{y^{\prime}}{\sigma_{b}{ }^{\prime}}=\frac{y}{\sigma_{b}} \quad$ or $\quad y^{\prime}=\frac{\sigma_{b}{ }^{\prime}}{\sigma_{b}} \times y$
Substituting the value of $y^{\prime}$ in equation (iv) we get

$$
\begin{aligned}
& \left(\frac{\sigma_{b}^{\prime}}{\sigma_{b}}\right)^{2} \times y=2 h+2 \frac{\sigma_{b}^{\prime}}{\sigma_{b}} \times y \\
& \text { or } \quad\left(\frac{\sigma_{b}^{\prime}}{\sigma_{b}}\right)^{2}-2 \frac{\sigma_{b}^{\prime}}{\sigma_{b}}-\frac{2 h}{y}=0
\end{aligned}
$$

Solving the above quadratic equation, we get

$$
\frac{\sigma_{b}^{\prime}}{\sigma_{b}}=\frac{2+\sqrt{2^{2}+4 \times \frac{2 h}{y}}}{2}=\left[1+\sqrt{1+\frac{2 h}{y}}\right]
$$

$\therefore \quad$ Impact stress due to bending $\sigma^{\prime}$

$$
=\sigma\left[1+\sqrt{1+\frac{2 h}{y}}\right] \quad E-2.41(P-20 A)
$$

Since the stress and deflections are proportional, the deflection of the beam due to impact is $y^{\prime}=y\left[1+\sqrt{1+\frac{2 h}{y}}\right] \quad \boldsymbol{E}-2.42(\boldsymbol{P}-20 A)$

The ratio of maximum stress to the static stress is called impact factor.
For bending loads, impact factor $=\frac{\sigma^{\prime}}{\sigma}$

$$
=1+\sqrt{1+\frac{2 h}{y}}
$$

## Torsional Impact:

In a similar manner, the equation for impact of a falling body on a torsion member can be found. The shear stress and angular deflection, due to impact at radius ' $r$ ' of a free falling load from a height ' $h$ ' as shown in fig are.


Impact stress due to torsion, $\tau^{\prime}=\tau\left[1+\sqrt{1+\frac{2 h}{r \theta}}\right] \boldsymbol{E}-2.42(\boldsymbol{P}-20 \boldsymbol{A})$
Since the shear stress and angular deflections are proportional, the deflection of the shaft due to impact is $\theta^{\prime}=\theta\left[1+\sqrt{1+\frac{2 h}{r \theta}}\right] \quad \boldsymbol{E}-2.42(\boldsymbol{P}-20 \boldsymbol{A})$
Where static angular deflections $(\theta)$ and impact angular deflections $\left(\theta^{\prime}\right)$ in radius, and ' $r$ ' is the moment arm of the load ' $W$ '.

## Impact factor;

The ratio of maximum shear stress to the static shear stress is called impact factor.
For Bending loads,

$$
\text { impact factor }=\frac{\tau^{\prime}}{\tau}=1+\sqrt{1+\frac{2 h}{r \theta}}
$$

## Effect of inertia:

When a body of weight ' $W$ ' strikes another body of weight W', some of the impact energy is used to overcome the inertia of weight W'. Hence the body is subjected to less impact and the resulting stresses and deformations in it are reduced. The stresses and deflections may be computed than one can be determined by the formula.

$$
n=\frac{1+a m}{(1+b m)^{2}} \quad \boldsymbol{E}-2.46(\boldsymbol{P}-20 \boldsymbol{A}) \quad \text { where } \quad m=\frac{W^{\prime}}{W}
$$

' $a$ ' and ' $b$ ' are constant coefficients whose values are given in Table $2.8 \mathbf{( P - 2 7 )}$ for the main cases of impact action.

## DESIGN FOR FATIGUE STRENGTH

## THEORY QUESTIONS:

1. What is S-N diagram?
(Dec.16/Jan. 17)(04 M)
2. Explain i) LCF ii) HCF
(Dec.16/Jan. 17)(04 M)
3. Fatigue stress concentration factor and notch sensitivity.
(June/July 2009)(04 Marks)
4. Explain stress versus number of cycles (S-N) curve for ferrous \& non ferrous metals with the aid of experimental sketch and characteristic curves.
(Dec.15/Jan.16)(06 Marks)
5. List the factors affecting the endurance limit.(June/ July 2016) (06 Marks)
6. Derive an expression for Goodman's relationship.
(June/ July 2014)(June/ July.2011) (Dec.2011)(06 Marks)
7. Derive Soderberg's relation for fluctuating loads.
(Dec.15/Jan.16)(06 Marks)
8. Derive Soderberg's relation for a member subjected to fatigue loading.
(May 2017)
9. Derive the Soderberg's relation (equation) for designing a machine element, with change in $\mathrm{c} / \mathrm{s}$, to sustain loads that fluctuate between two limits, taking the stress concentration into account.
(May/June.2010)(Dec.2010)(Dec.09/Jan.10)(Dec.08/Jan.09)(06 M)
10. Explain the following:
i) Endurance limit (Strength) ii) HCF \& LCF
(Dec.16/Jan. 17)(04 M)

## NUMERICAL PROBLEMS:

1. A steel rod $\sigma_{u}=1089.5 \mathrm{MPa} ; \sigma_{y}=689.4 \mathrm{MPa} ; \sigma_{e n}=427.6 \mathrm{MPa}$ is subjected to a tensile load with varies from120 kN to 40 kN . Design the safe diameter of the rod using "Soderberg's diagram". Adopt factor of safety as 2, stress concentration factor as unity and correction factors for load, size and surface as $0.75,0.85$ and 0.91 respectively.
(Dec.07/ Jan.08)
2. A steel rod (SAE 9260 oil quenched) is subjected to a tensile load with varies from 120 kN to 40 kN . Design the safe diameter of the rod using "Soderberg's diagram". Adopt factor of safety as 2.
3. A SAE 1025 annealed steel rod of circular cross section is subjected to completely reverse bending moment of $500 N-m$. Determine the diameter of the required based on a factor of safety of 2. $\sigma_{y}=234.4 \mathrm{MPa}, \sigma_{e n}=$ 200.1 MPa , size factor $=0.9$, surface factor $=0.85$, load factor $=1$.
(Dec.16/Jan.17)(marks 10)
4. A rough finished steel rod having $\sigma_{u}=620 \mathrm{MPa}, \sigma_{y}=400 \mathrm{MPa}, \sigma_{e n}=$ 345 MPa is subjected to completely reverse bending moment of $400 \mathrm{~N}-\mathrm{m}$. Determine the diameter of the required based on a factor of safety of 2.5.
(Dec.08/Jan.09)(marks 12)
5. A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN . Determine the diameter of bar by taking a factor of safety of 2 , size effect of 0.85 , surface finish factor of 0.9 . The material properties of bar are given by $\sigma_{u}=650 \mathrm{MPa}, \sigma_{y}=500 \mathrm{MPa}, \sigma_{e n}=$ 350 MPa .
(June/July 2011)(Marks 12)
6. A steeped shaft of circular cross section shown in below fig is subjected to variable load which is completely reversed with a value equal to 100 kN . It is made of SAE 1045 steel annealed. Determine the diameter ' $d$ ' and ' $r$ ', so that the maximum stress will be limited to a value corresponding to a factor of safety of 2 . Notch sensitivity index $=1$.

7. A steeped shaft of circular cross section shown in Fig. is made of SAE 1045 annealed steel. The load is repeated and completely reversed with a value of 100000 N . taking $\frac{r}{d}=\frac{1}{8}$, determine the diameter ' $d$ ' and the fillet radius ' $r$ ' so that the maximum stress will be limited to a value corresponding to a factor of safety 2 . Consider the load factor $=1$, surface finish factor $=0.85$ and size factor $=0.9$.
(Dec.15/Jan.16)(marks 08)

8. A shaft supported by bearings 400 mm apart is subjected to a concentrated load varying from " $W$ " to " $3 W$ " at its mid point. The shaft is 50 mm diameter. Estimate the value of " $W$ " with a factor of safety of 1.5 . The material has an ultimate strength of 700 MPa , endurance limit of 350 MPa and yield strength of 525 MPa . Take size factor of 0.85 and a surface finish factor of 0.848 .
(Dec.13/ Jan.14) (14 Marks)
9. A simply supported beam has a concentrated load at the center which fluctuates from a value of P to 4 P . The span of the beam is 50 mm and its cross section is circular with a diameter of 60 mm . taking for the beam material an ultimate stress of 700 MPa , a yield stress of 500 MPa , endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3. Calculate the maximum load of P. take a size factor of 0.85 , a surface finish factor of 0.9 and fatigue stress concentrations factor of 1 .
(Dec.16/ Jan.17) (16 Marks)
10. A cantilever beam made of $35 C 8$ steel $\left(\sigma_{u t}=540 \mathrm{MPa}\right)$ is subjected to a completely reversed load of 1000 N as shown in Fig. the notch sensitivity factor ' $q$ ' at the fillet can be taken as 0.85 and expected reliability of $90 \%$. Determine the diameter of the beam for life cycles of 10000 cycles. (June 2012)(10 M)

11. A component machined from a plate made of steel 45C8 ( $\sigma_{u t}=$ 630 MPa ) is shown in Fig. it is subjected to a completely to a completely reversed axial force of 50 kN . The expected reliability is $90 \%$ and the factor of safety is 2 . Determine the plate thickness ' $t$ ' for infinite life, if the notch sensitivity factor is 0.8 .
(Dec.08/Jan.09)(12 M)

12. A cast iron shaft with an ultimate strength of 180 MPa is subjected to a torsional load which is completely reversal. The load is to be applied an indefinite number of cycles. If the shaft is 50 mm diameter and is joined with 75 m diameter shaft with a billed radius of 12.5 mm . use factor of safety 2 . What is the maximum torque that can be applied to the shaft? Take surface factor $=0.75$, size factor $=0.85$, load factor $=1 . \quad$ (June\July 2013)(10 M)
13. A stepped shaft with a reduction ratio of 1.2 is to have a fillet radius of $10 \%$ of smaller diameter. This shaft is to be made of a material that has a notch sensitivity factor of 0.925 . A shear stress of 160 MPa at yield and a shear stress of $120 M P a$ at endurance limit. The surface of the component would have a surface factor of 0.95 , and the size factor is 0.85 . Determine the diameter at the minimum cross - section to sustain a twisting moment that fluctuates between 500 Nm and 800 Nm . Assume $\mathrm{FoS}=2$.
(May/June 2010)(14 Marks)
14. A 40 mm diameter steel shaft has $\sigma_{\mathrm{y}}=413 \mathrm{MPa}, \sigma_{\mathrm{en}}=336 \mathrm{MPa}$. for a factor of safety of $n=2$. What
i) Repeated
ii) Reversed, torques can the shaft sustain indefinitely. The shaft has a groove machined on it the radius of the groove is 2 mm and the diameter at the bottom of the groove is 36 mm . take size factor $=0.85$, surface factor $=1$.
(May 2017)(14 Marks)
15. A cantilever shaft is subjected to a load of $F$ Newton (up) and $3 F$ Newton (down). The shaft is steeped down from 20 mm diameter to 13 mm diameter and is provided with a fillet of radius 3 mm . The length of the shaft is 150 mm . The fillet section is at a distance of 25 mm from the fixed end. Determine the maximum load that the member can withstand for infinite life. Assume $F o S=2 ; \sigma_{u l t}=$ $550 \mathrm{MPa}, \sigma_{y i d}=470 M P a, q=0.9$, compare the value of load by both Goodman and Soderberg's criteria.

## (June/July 2017)(Dec.09/ Jan.10) (July 2007) (June 2012)

 (Dec.2010)(15 Marks)16. A $S A E 1025$ water quenched steel $\operatorname{rod}\left(\sigma_{u}=620.8 M P a, \sigma_{y}=\right.$ 400.1 MPa, $\left.\sigma_{e n}=345.2 M P a\right)$ of circular cross section. Shown in Fig, is subjected to load varying from $P$ to $3 P$. Determine the value of ${ }^{\prime} P^{\prime}$. The stress concentration factor may be taken as 1.4 . Analyze the member at the change of cross section. Use factor of safety $=3$. Assume the surface to be rough finished.
(Dec.06/ Jan.07) (15 Marks)

17. A $S A E 1025$ water quenched steel $\operatorname{rod}\left(\sigma_{u}=600 M P a, \sigma_{y}=\right.$ $390 \mathrm{MPa}, \sigma_{\text {en }}=320 \mathrm{MPa}$ ) of circular cross section. Shown in Fig, is subjected to load varying from $P$ upward to $3 P$ downward. Determine the value of ${ }^{\prime} P^{\prime}$. Taking factor of safety as 3 . Analyze the member at the change of cross section. The size and surface factor may may be taken as 0.85 and 0.9 respectively.

(June/July 2014) (15 Marks)
18. A cold drawn steel rod of circular section is subjected to variable bending moment of 565 Nm to 1130 Nm as the axial load varies from 4500 N to 13500 N . the maximum bending moment occurs at the same instant that the axial load is maximum. Determine the required diameter of the rod for a factor of safety $=2$. Neglect stress concentration and column effect. Take $\sigma_{u}=550 \mathrm{MPa}, \sigma_{y t}=470 \mathrm{MPa}$, endurance limit as $50 \%$ of the ultimate strength and size, load and surface correction coefficients as $0.85,1$ and 0.85 respectively.
(Dec.09/Jan.10)(14 M)
19. A steel member of circular section, is subjected to a torsional shear stress that varies from 0 to 35 MPa . At same time, it is subjected to an axial stress that varies from -14 MPa to +28 MPa . Neglecting stress concentration and column effect and assuming that the maximum stress in torsional and axial load occur at the same time, determine i) the maximum equivalent shear stress, ii) the design factor of safety based upon yield in shear. The material has an endurance limit $\sigma_{-1}=206 \mathrm{MPa}$, and a yield strength $\sigma_{y t}=480 \mathrm{MPa}$. The size factor $=1$, the surface has a mirror polish $=1$.
(Dec. 2011)(15 M)
20. A hot rolled steel shaft is subjected to a torsional load that varies from $330 N-m$ (clockwise) to $110 N-m$ (counter clockwise) as on applied bending moment at a critical section varies from $+440 \mathrm{~N}-\mathrm{m}$ to $-220 \mathrm{~N}-$ $m$. The shaft is of uniform cross section and no key way is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of $550 \mathrm{~N} / \mathrm{mm}^{2}$ and yield strength of $410 \mathrm{~N} / \mathrm{mm}^{2}$. Base design on a factor of safety $=1.5$. Take the endurance limit as half the ultimate strength, size factor of 0.85 and a surface finish factor of 0.62 .neglect the effect of stress concentration.(June/July 2014) (Dec 2010) (May/June 2010)
21. A pulley is mounted on a shaft midway between two bearings. The bending moment at the pulley varies from $200 N-m$, to $600 N-m$ and the torsional moment varies from $60 N-m$ to $180 N-m$. The frequency of variation of loads is the same as that the shaft speeds. The shaft is made of cold drawn steel having an ultimate strength of 550 MPa and yield strength of 400 MPa . Determine the required diameter of the shaft for in definite life. The stress concentration factor for the keyway present in bending and torsion may be taken as 1.5 and 1.2 respectively. Assume a factor of safety of 1.5 .

## (June/July 2009) ( $\mathbf{1 5}$ Marks)

22. A round rod of diameter $1.2 d$ has a semicircular groove of diameter of 0.2 d . This rod is to sustain a twisting moment that fluctuates between $2.5 \mathrm{kN}-\mathrm{m}$ and $1.5 \mathrm{kN}-\mathrm{m}$ together with a bending moment that fluctuates between $+2 k N-m$ and $-1 k N-m$. Selecting carbon steel C30 as material for the rod and choosing 2.5 as value for the factor of safety, determine a safe value for ' $d$ '.
(July 2006) ( 15 Marks)
23. A transmission shaft carries a pulley midway between two bearings. The bending moment at the pulley varies from $200 N-m$, to $600 N-m$ and the torsional moment varies from $70 N-m$ to $200 N-m$. The frequencies of variation of bending and torsional moment are equal to the shaft speeds. The shaft is made of cold drawn steel having an ultimate strength of 540 MPa and yield strength of 400 MPa . The correct endurance strength of the shaft is 200 MPa . Determine the required diameter of the shaft using a factor of safety of 2 .
(Dec.14/Jan 2015)( 20 Marks)
24. A hot rolled steel shaft is subjected to a torsional load that varies from $300 \mathrm{~N}-\mathrm{m}$ clockwise to $100 \mathrm{~N}-\mathrm{m}$ counter clockwise as on applied bending moment at a critical section varies from+400 $N-m$ to $-200 N-$ $m$. The shaft is of uniform cross section and no key way is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MPa and yield strength of 410 MPa . Base design on a factor of safety $=2$. Take the endurance limit as half the ultimate strength. Assume surface, size and load factor for bending as $1.111,1.1765$ and 1 and that for torsion as $1.05263,1.1765$ and 1.7 respectively.
(June/July 2013)(15 Marks)
25. A hot rolled steel shaft is subjected to a torsional load that varies from $330 N-m$ clockwise to $110 N-m$ counter clockwise as on applied bending moment at a critical section varies from $+440 \mathrm{~N}-\mathrm{m}$ to $-220 \mathrm{~N}-$ $m$. The shaft is of uniform cross section and no key way is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of $550 \mathrm{~N} / \mathrm{mm}^{2}$ and yield strength of $410 \mathrm{~N} / \mathrm{mm}^{2}$. Base design on a factor of safety $=1.5$. Take the endurance limit as half the ultimate strength.
(June/July 2009)(Dec.08/ Jan.09) (15 Marks)
26. A machine component shown in Fig is loaded as indicated on the diagram. Determine cross section dimension ' $d$ ' for the component, its fatigue stress concentration factor in bending and axial loading are 1.8 and 1.6 respectively. The allowable stress for the component is 100 MPa , the material properties are $\sigma_{u}=40 M P a, \sigma_{y}=250 M P a$

27. A cold drawn steel cantilever member shown in below fig is subjected to a transverse load and its end that varies from 50 N (up) to 150 N (down) as an axial load varies from 100 N compression to 500 N tension. Determine the required diameter of the section for infinite life using a factor of safety of2. The material has an $\sigma_{u}=550 M P a, \sigma_{y}=470 M P a$ take a notch sensitivity factor for the fillet as 0.9.
(Dec.2011)(Dec.09/Jan.10)(20 Marks)

28. A steel cantilever member shown in below fig is subjected to a transverse load and its end that varies from $45 N(u p)$ to $135 N$ (down) as an axial load varies from 110 N compression to 450 N tension. Determine the required diameter at the change of section for infinite life using a factor of safety of 2. The strength properties of the material are $\sigma_{u}=550 M P a, \sigma_{y}=470 M P a$ and $\sigma_{e n}=275 M P a$. notch sensitivity index $\mathrm{q}=1$.
29. Determine the diameter ' $d$ ' based on Soderberg's criterion for a machine member as shown in the Fig. the properties of the material used are, $\sigma_{u}=$ $600 \mathrm{MPa}, \sigma_{y}=400 \mathrm{MPa}, \sigma_{e n}=300 \mathrm{MPa}, \tau_{y}=200 \mathrm{MPa}$. Assume size, surface and load factor for bending as $0.9,0.85$ and 1 and that for axial as $0.9,0.85$ and 0.6 respectively. The notch sensitivity factor is 0.92 and factor of safety as 3 .
(June / July 2016)( (14 Marks)

30. A ground steel cantilever member shown in below fig is subjected to a transverse load and its end that varies from $100 N(u p)$ to $200 N$ (down) as an axial load varies from500 N compression to 1000 N tension. Determine the required diameter of the section using a factor of safety of 2 . The strength properties of the material are $\sigma_{u}=550 \mathrm{MPa}, \sigma_{y}=480 \mathrm{MPa}$ and $\sigma_{e n}=$ $270 M P a$. notch sensitivity index $q=1$.

(Dec.09/Jan. 10)(20 Marks)
31. A machine component is subjected to a tensile stress in $X$ - direction which varies from 40 to 100 MPa . A tensile stress applied in Y - direction varies from 10 to 80 MPa . Frequency of variation of these stresses is equal. The corrected value of endurance limit of the component is 270 MPa . Ultimate tensile strength is 660 MPa . Determine the factor of safety of this component under fatigue loading.

## Cumulative damage in fatigue (miner's equation)

32. The work cycle of a mechanical component subjected to completely reversed bending stress consists of the following three element:
i. $\pm 350 \mathrm{MPa}$ for $85 \%$ of time.
ii. $\pm 400 \mathrm{MPa}$ for $12 \%$ of time.
iii. $\pm 500 \mathrm{MPa}$ for $3 \%$ of time.

The material for the component is $50 \mathrm{C} 4\left(\sigma_{u}=660 \mathrm{~N} / \mathrm{mm}^{2}\right)$ and the corrected endurance strength of the component is 280 MPa . Determine the life of the component.
(Dec. 2012)(Marks 14)
33. A machine member made of SAE 1095 steel (oil Quenched) is subjected to reversed stress cycles. If the effective endurance strength of the component is 600 rpm , compute the cycles of life for the machine member. element:
iv. $70 \%$ of time with 650 MPa .
v. $15 \%$ of time with 700 MPa .
vi. $10 \%$ of time with 750 MPa .
vii. $5 \%$ of time with 800 MPa .
(Dec. 2010)(12 M)

## DESIGN FOR FATIGUE STRENGTH:

## Introduction:

In the previous topics, the forces acting externally on a machine component were assumed to be static. In many applications, the components are subjected to forces which are not static, but vary in magnitude with respect to time. Static or quasi-static (gradually applied) loads are rarely observed in machines. Parts of reciprocating machines, cam shafts, shafts carrying gears, sprockets, pulleys and rotors not properly balanced, and gear teeth are a few examples of parts subjected to cyclic or fluctuating loads. Most of the failures and accidents encountered are due to fluctuating and suddenly applied loads.

## Type of applied load

i) Static load

Does not change in magnitude and direction and normally gradually to a static value.
ii) Variable load

- Change in magnitude
- Traffic of varying weight passing bridge
- Change in direction
- Load on piston rod of double acting cylinder


Static Load


Variable Load

## Explain the following:

## i) Endurance limit (Strength) (Dec.16/Jan. 17)(04 M)

## Fatigue Loading:

Fatigue is a time varying load acting on a component or a system when loads are stressed on a $\mathrm{m} / \mathrm{c}$ part changed in magnitude or direction or both loading is known as visible or fatigue loading.

Fatigue assumes importance because fatigue failure in member can occur at stress values that are below yield strength or ultimate strength of material is most often eases.

It has been observed that some of the machine parts which are subjected to variable loading fail at a stress value lower than that of the ultimate or yield strength of material. The stresses induced due to such forces are called as fluctuating stresses; such failure of the part is generally termed fatigue failure. It is observed that about $50 \%$ of failures of mechanical components are due to fatigue resulting from fluctuating stresses. To the purpose of design analysis, simple models for stress-time relationship are used.

The most popular model for stress time relationship is the sine curve as shown in below Figures (a) and (b). $\sigma_{\text {Max }}$ and $\sigma_{\text {Min }}$ are maximum and minimum stresses, while $\sigma_{m}$ and $\sigma_{a}$, are called mean stress and stress amplitude respectively.

It can be proved that

$$
\sigma_{m}=\frac{1}{2}\left(\sigma_{\text {Max }}+\sigma_{\text {Min }}\right) \quad \sigma_{a}=\frac{1}{2}\left(\sigma_{\text {Max }}-\sigma_{\text {Min }}\right)
$$

## Terminologies in fatigue loading:

Amplitude stress, $\sigma_{a}=\frac{\sigma_{\max }-\sigma_{\min }}{2}$, Mean stress, $\quad \sigma_{a}=\frac{\sigma_{\max }+\sigma_{\min }}{2}$,

$$
\text { Stress } \operatorname{range}(\Delta \sigma), \quad \Delta \sigma=\sigma_{\max }-\sigma_{\min }
$$

Fully Reverse Loading: Stress which vary from one value of compression stress to

Some value of tensile stress or vice
 versa.

$$
\begin{aligned}
\sigma_{m} & =\frac{\sigma_{\max }+\sigma_{\min }}{2}=0 \\
\sigma_{a} & =\frac{\sigma_{\max -\sigma_{\min }}^{2}=\sigma_{\max }}{} \\
\Delta \sigma & =\sigma_{\max }-\sigma_{\min }
\end{aligned}
$$

Repeated Loading: The stress which vary from zero to certain maximum value (tensile or compressive nature) are called repeated loading.


$$
\begin{gathered}
\sigma_{m}=\frac{\sigma_{\max }+\sigma_{\min }}{2}=\frac{\sigma_{\max }}{2} \\
\sigma_{a}=\frac{\sigma_{\max }-\sigma_{\min }}{2}=\frac{\sigma_{\max }}{2}
\end{gathered}
$$

## Fluctuating Loading:

The stress which vary from minimum value to maximum value of same nature (compressive or tensile) is called fluctuating stress.


$$
\begin{aligned}
\sigma_{a} & =\frac{\sigma_{\max }-\sigma_{\min }}{2} \\
\sigma_{m} & =\frac{\sigma_{\max }+\sigma_{\min }}{2} \\
\Delta \sigma & =\sigma_{\max }-\sigma_{\min }
\end{aligned}
$$

## Alternating Loading

The stress which vary from one magnitude in one direction and another magnitude in other direction (compressive or tensile) is called alternating stress.


## Material property under variable load

- Specimens are subjected to repeated or varying loads of specific magnitude
- No of cycles required for fracture is recorded.
- Above two steps are repeated for different load magnitude.


## Fatigue Curve


c) Typical Test Specimen


Rotating Beam Test (Constant B.M Type)

## What is S-N diagram?

(Dec.16/Jan. 17)(04 M)
S-N diagram defines te number of cycles to failure ( N ), when a material is repeatedly cycled through a given stress range ( S )

The number of cycles on the log-log graph as shown in below Figure. This curve is called the fatigue curve.


Fatigue Curve for Steel

To develop on S-N plot in fatigue laboratory by standard methods. The procedure to be followed.

1. Select a large group of carefully prepared polished fatigue specimen of diameter 7.5 mm of the material of interest.
2. Select 6 or 8 stress level.
3. To make each test run, mount a specimen in the rotating beam testing machine using due care. Set the machine for the desired stress amplitude with cycle counter set on zero.
4. In first test, Start the machine and run at constant stress lower than the ultimate strength of the material until the specimen fails.
5. Record the stress amplitude used \& the cycle count at the time of failure or run out.
6. Using new specimen, In the second test, specimens are tested with a stress slightly less than that used for first test and again the number of cycles before the fracture takes place is counted.
7. Change the new stress level less than previous stress level and repeat the procedure until all specimens have been tested.
8. Note that entire output from a complete fatigue test is a single point on S-N plot.
9. Lot all data collected on a stress $\mathrm{v} / \mathrm{s}$ life coordinate system. Run outs or points for which fatigue failure was observed during the test are indicated dot points.

## Explain i) LCF ii) HCF

(Dec.16/Jan. 17)(04 M)
the specimens survived for less than 1000 cycles. It is called the low cycle area or infant mortality period. However, the specimens which fail after attaining at- least 1000 cycles are classified as high cycle fatigue.

| Low cycle fatigue | The specimens survived for less than 1000 cycles |
| :--- | :--- | :--- |
| High cycle fatigue | The specimens which fail after attaining at least 1000 cycles |

Since fatigue failure is statistical in nature and depends upon the type of stress and the material of the specimen, it is generally found that $\sigma-N$ curves of different shapes are obtained.

In the case of plain carbon steel, a clear sharp corner called the knee occurs at about $10^{6}$ cycles as shown in the graph. Beyond the knee point, the trend of the curve is almost asymptotic to abscissa which indicates that beyond this point failure will not occur no matter how great the number of cycles. The strength corresponding to the knee point is called the endurance strength ( $\sigma_{e n}$ ) or fatigue limit or endurance limit. For non-ferrous metals and other ferrous metals such as cast iron this curve never becomes horizontal [see Figure (d).]. Hence, these materials do not have any specific endurance limit.

As mentioned earlier, the endurance, limit depend upon the type of load, the stress, and the material of the component. Different materials when subjected to different types of load exhibit different endurance limits. Generally, it is observed that the materials which have better static properties are also good under fatigue loading, although no exact relationship between the ultimate strength and the endurance limit exists. For preliminary design calculations, the following relations are quite useful.

## 1. Material considerations:

For steel $\quad \sigma_{e n}=0.5 \sigma_{u} \quad(\boldsymbol{E}-\mathbf{2 . 1 7 a})(\boldsymbol{P}-\mathbf{1 7})$
For cyclic torsion load: $\quad \tau_{e n}=(0.5) \sigma_{e n}$ for steel $\quad(\mathbf{E}-\mathbf{2 . 1 9 a})(\mathbf{P}-\mathbf{1 7})$

## FATIGUE OR ENDURANCE LIMIT

The fatigue or endurance limit ( $\sigma_{e n}$ )for material is defined as the maximum value of completely reversing stress that the standard specimen can sustain for infinite number of cycles without fatigue failure.


For a stress below a certain value, the material will not fail whatever be no. of cycles represented by dotted line is known as endurance or fatigue limit.

## FATIGUE STRESS CONCENTRATION FACTOR AND NOTCH SENSITIVITY (June/July 2009)(04 Marks)

It is observed that the actual reduction in the endurance strength of a material due to stress concentration is less than the amount indicated by the theoretical stress concentration factor $K_{t}$ two separate notations $K_{t}$, and $K_{t f}$ are therefore used for stress concentration factors. $K_{t}$ is the theoretical stress concentration factor, as defined in previous section, which is applicable to ideal materials that are homogeneous, isotropic and elastic. $K_{t f}$, is the fatigue stress concentration factor, which is defined as follows:

$$
K_{t f}=\frac{\text { Endurance limit of the notch }- \text { free specimen }}{\text { Endurance limit of the notched specimen }}
$$

Notch Sensitivity ( $\mathbf{q}$ ) is defined as the susceptibility of a material to succumb to the damaging effects of stress raising notches in fatigue loading. The notch sensitivity factor $q$ is defined as

$$
\begin{gathered}
q=\frac{\text { Increase of actual stress over nominal stress }}{\text { Increase of theoretical stress over nominal stress }} \\
\qquad q=\frac{K_{t f}-1}{K_{t}-1}
\end{gathered}
$$

The; above equation is rearranged in the following form
For normal stress

$$
\begin{equation*}
K_{t f}=1+q\left(K_{t}-1\right) \tag{E2.5}
\end{equation*}
$$

For shear stress

$$
\begin{equation*}
K_{t s f}=1+q\left(K_{t s}-1\right) \tag{E2.7}
\end{equation*}
$$

This equations shows that when $q=0\left(K_{t f}\right.$ or $\left.K_{t s f}=1\right)$, the material has no sensitivity to notches. When $q=1\left(K_{t f}=K_{t}\right.$ and $\left.K_{t s f}=K_{t s}\right)$. The material has full sensitivity to notches. In case of doubt, the designer should use $q=0\left(K_{t f}\right.$ or $\left.K_{t s f}=1\right)$ and the design will be on safe side.

## FACTORS THAT AFFECT FATIGUE STRENGTH

The fatigue strength of a part made of a given material strongly depends on

1. Type of loading,
2. Absolute dimensions of a part
3. Surface condition and corrosion,
4. Geometrical discontinuities
(Axial, Torsion, Bending) (Load correction factor)(A) (Size correction Factor) (B)
(Surface correction Factor) (C) (holes, notches, fillets etc.)
(Stress concentration Factor)( $K_{t f}$ or $K_{t s f}$ )
5. Grain size and direction,
6. Operating temperature,
7. Heat-treatment,
8. Residual stresses, Type of cycle (degree of its asymmetry).

Some of the above factors are being discussed in detail in the following articles.
Load correction factor
Reversed bending load

| Reversed axial load | $\mathrm{A}=0.7$ to 1 | Take $\mathrm{A}=0.7$ |
| :--- | :--- | :--- |
| Reversed torsional load | $\mathrm{A}=0.5$ to 0.6 | Take $\mathrm{A}=0.6$ |

## Size correction factor (B)

The endurance limit of the component is found to decrease with the increasing size of the specimen when it is subjected to bending and torsion loading, because an increase in the size of a specimen is apt to have more internal defects.

The value of the size factor ' $B$ ' for a round component is calculated by the following empirical relations:

| Diameter (d), mm | Size correction factor (B) |
| :---: | :--- |
| $d \leq 7.5 \mathrm{~mm}$ | 1.00 |
| $7.5<d \leq 50 \mathrm{~mm}$ | 0.85 |
| $d>50 \mathrm{~mm}$ | 0.75 |

Where ' $d$ ' is the diameter of the specimen.
In rectangular components, first an equivalent diameter is calculated by the constant volume approach and then the corresponding size factor is determined.

## Surface correction factor (C) [(P-24)(T-2.2)] [(P-40)(Fig - 2.26)]

The surface conditions of machine parts vary with the type of machining or shaping operations that are performed upon them. Experiments have shown that parts with poor surface finish have reduced fatigue limit compared to parts with smooth surface finish. The reduction in fatigue limit is due to signs of tool marks left on the surface.


The degree of smoothness of the surface and the endurance limit reduce in the following order of operations: (i) polishing, (ii) grinding.(iii) Fine turning, and (iv) rough tuning. Above Figure gives the value of the surface finish factor for a known value of the ultimate strength and the type of surface finishing operation.

## FATIGUE DESIGN APPROACHES:

For designing a machine part under these stress cycles, generally, two approaches called (i) the Goodman's approach and (ii) the Soderberg's approach are most commonly used. The Goodman's approach is based on ultimate strength of material whereas the Soderberg's approach uses yield strength. Experimentally it is established that in fatigue design of the part for fluctuating stresses, the Goodman's approach is more accurate while the Soderberg's approach is more conservative.

A brief discussion on these design approaches is given below:

1. Derive an expression for Goodman's relationship.
(June/ July.2011) (Dec.2011)(06 Marks)

## Goodman's Relationship

According to the Goodman's approach, the design of a component subjected to fluctuating stress can be carried out by the following steps (Figure (f)).

(f). Goodman's Relationship

In deriving Goodman's equation, endurance stress $\sigma_{e n}$ is plotted along Y-axis and yield stress along X - axis.

OA and OB represent the endurance stress and ultimate stress respectively. The line AB represents the Goodman's failure line. Using factor of safety n , line CD is drawn parallel to AB represents safe stress line A point ' P ' on CD has variable stress $\sigma_{a}$ on Y -axis and mean stress $\sigma_{m}$ on X-axis.

From similar triangles COD and PED,

$$
\begin{gather*}
\frac{C O}{P E}=\frac{O D}{E D} \quad \ldots \ldots(1)  \tag{1}\\
P E=\sigma_{a} \\
O D=\frac{\sigma_{u}}{n}
\end{gather*} \quad \text { Where } C O=\frac{\sigma_{e n}}{n}
$$

And $\quad \mathrm{ED}=\mathrm{OD}-\mathrm{OE}=\frac{\sigma_{u}}{n}-\sigma_{m}$
Substituting in equation (1) we get

$$
\begin{gathered}
\frac{\left(\frac{\sigma_{e n}}{n}\right)}{\sigma_{a}}=\frac{\left(\frac{\sigma_{u}}{n}\right)}{\frac{\sigma_{u}}{n}-\sigma_{m}} \\
\frac{\sigma_{a}}{\frac{\sigma_{e n}}{n}}=\frac{\frac{\sigma_{u}}{n}}{\frac{\sigma_{u}}{n}}-\frac{\sigma_{m}}{\frac{\sigma_{u}}{n}} \\
\frac{\sigma_{a}}{\frac{\sigma_{e n}}{n}}=1-\frac{\sigma_{m}}{\frac{\sigma_{u}}{n}} \\
\frac{\sigma_{a}}{\sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{u}}=\frac{1}{n}
\end{gathered}
$$

Goodman's relation for fluctuating normal stress

$$
\begin{equation*}
\frac{\sigma_{a}}{\sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{u}}=\frac{1}{n} \tag{E2.27}
\end{equation*}
$$

Goodman's relation for fluctuating torsional stress.

$$
\begin{equation*}
\frac{\tau_{a}}{\tau_{e n}}+\frac{\tau_{m}}{\tau_{u}}=\frac{1}{n} \tag{E2.29}
\end{equation*}
$$

Considering stress concentration factor ( $K_{t f}$ ) and important three modifying (correction ) ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$ ) factors for the endurance limit,

The Goodman's equation for ductile material:

$$
\begin{equation*}
\frac{K_{t f} \sigma_{a}}{A B C \sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{u}}=\frac{1}{n} \tag{E2.32a}
\end{equation*}
$$

2. Derive the Soderberg's relation (equation) for designing a machine element, with change in $\mathrm{c} / \mathrm{s}$, to sustain loads that fluctuate between two limits, taking the stress concentration into account.
3. Derive Soderberg's relation for a member subjected to fatigue loading.
(May 2017)(Dec.16/Jan.17) (May/June.2010)
(Dec.09/Jan.10) (Dec.08/Jan.09) (06 Marks)

## Soderberg's relationship

According to the Soderberg's approach, the design of a component subjected to fluctuating stress can be carried out by the following steps (Figure (g)).


In deriving Soderberg's equation, endurance stress $\sigma_{e n}$ is plotted along Y-axis and yield stress along X - axis.

OA and OB represent the endurance stress and yield stress respectively. The line AB represents the Soderberg's failure line. Using factor of safety $n$, line CD is drawn parallel to AB represents safe stress line A point ' P ' on CD has variable stress $\sigma_{a}$ on Y -axis and mean stress $\sigma_{m}$ on X-axis.

From similar triangles COD and PED,

$$
\begin{equation*}
\frac{C O}{P E}=\frac{O D}{E D} \tag{1}
\end{equation*}
$$

Where $C O=\frac{\sigma_{e n}}{n}$

$$
P E=\sigma_{a} ; \quad O D=\frac{\sigma_{y}}{n}
$$

And $\quad \mathrm{ED}=\mathrm{OD}-\mathrm{OE}=\frac{\sigma_{y}}{n}-\sigma_{m}$
Substituting in equation (1) we get

$$
\begin{aligned}
& \frac{\left(\frac{\sigma_{e n}}{n}\right)}{\sigma_{a}}=\frac{\left(\frac{\sigma_{y}}{n}\right)}{\frac{\sigma_{y}}{n}-\sigma_{m}} \\
& \frac{\sigma_{a}}{\frac{\sigma_{e n}}{n}}=\frac{\frac{\sigma_{y}}{n}}{\frac{\sigma_{y}}{n}}-\frac{\sigma_{m}}{\frac{\sigma_{y}}{n}} \\
& \frac{\sigma_{a}}{\frac{\sigma_{e n}}{n}}=1-\frac{\sigma_{m}}{\frac{\sigma_{y}}{n}} \\
& \frac{\sigma_{a}}{\sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{y}}=\frac{1}{n}
\end{aligned}
$$

Soderberg's relation for fluctuating normal stress

$$
\begin{equation*}
\frac{\sigma_{a}}{\sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{y}}=\frac{1}{n} \tag{E2.28}
\end{equation*}
$$

Soderberg's relation for fluctuating torsional stress

$$
\frac{\tau_{a}}{\tau_{e n}}+\frac{\tau_{m}}{\tau_{y}}=\frac{1}{n}
$$

$$
(E 2.30) P-18
$$

Considering stress concentration factor ( $K_{t f}$ ) and important three modifying (correction) ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$ ) factors for the endurance limit,

The Soderberg's equation for ductile material in normal stress:

$$
\begin{equation*}
\frac{K_{t f} \sigma_{a}}{A B C \sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{y}}=\frac{1}{n} \tag{E2.33a}
\end{equation*}
$$

The Soderberg's equation for ductile material in shear stress:

$$
\begin{equation*}
\frac{K_{t f} \sigma_{a}}{A B C \sigma_{e n}}+\frac{\sigma_{m}}{\sigma_{y}}=\frac{1}{n} \tag{E2.33c}
\end{equation*}
$$

## Cumulative damage in fatigue (miner's equation)

When more than one stress level above the endurance limit is present in working cycle, it becomes necessary to combine the effects of various stresses to obtain an overall estimate of the expected life.

Suppose at stress level $\sigma_{1}$, the fatigue life is $N_{1}{ }^{\prime}$ cycles $\&$ the component actually runs for $N^{\prime}{ }_{1}$ cycles \& at stress level $\sigma_{2}$, the fatigue life is $N_{2}{ }^{\prime}$ cycles \& the component actually runs for $N^{\prime}{ }_{2}$ cycles.

The miner's equation is given by

$$
\frac{N_{1}^{\prime}}{N_{1}}+\frac{N_{2}^{\prime}}{N_{2}}+----+\frac{N_{n}^{\prime}{ }_{n}}{N_{n}}=1
$$

If $N_{C}$ is the combined life
$\div$ both sides of above equation by $N_{C}$

$$
\begin{gather*}
\frac{N_{1}^{\prime}}{N_{1} N_{C}}+\frac{N_{2}^{\prime}}{N_{2} N_{C}}+---+\frac{N_{n}^{\prime}}{N_{n} N_{C}}=\frac{1}{N_{C}} \\
\frac{\alpha_{1}}{N_{f 1}}+\frac{\alpha_{1}}{N_{f 2}}+----+\frac{\alpha_{1}}{N_{f n}}=\frac{1}{N_{C}} \tag{or}
\end{gather*}
$$

Where $\alpha_{1}=\frac{N^{\prime}{ }_{i}}{N_{C}}$
The miner's equation can be used only for complete reversal of stresses.

## Module - 3

## Design of Shafts, Joints, Couplings and Keys:

Torsion of shafts, design for strength and rigidity with steady loading, ASME codes for power transmission shafting, shafts under combined loads.

Design of Cotter and Knuckle joints, Rigid and flexible couplings, Flange coupling, Bush and Pin type coupling and Oldham's coupling. Design of keys-square, saddle, flat and father.

## KEYS:

## THEORY QUESTIONS:

1. Prove that a square key is equally strong in crushing and in shear.
(May 2017)(04 M)
2. If a shaft and key are made of same material, determine the length of the key required in terms of shaft diameter, taking key width $b=d / 4$ and the key thickness $h=3 d / 16$.

## NUMERICAL PROBLEMS:

3. A standard cross section of a flat key, which is fitted on a 50 mmdiameter shaft, is $16 \times 10 \mathrm{~mm}$. The key is transmitting $475 \mathrm{~N}-\mathrm{m}$ torque from the shaft to the hub. The key is made of commercial steel for which yield strength in both tension and compression may be taken as 230 MPa . Determine the minimum length of key required if the factor of safety is 3 .
(June 2012) (06 Marks)
4. A rectangular sunk key 14 mm wide $\times 10 \mathrm{~mm}$ thick $\times 75 \mathrm{~mm}$ long is required to transmit $1200 \mathrm{~N}-\mathrm{m}$ torque from a 50 mm diameter solid shaft. Determine whether the length is sufficient or not if the permissible shear stress and crushing stress are limited to 56 MPa and 168 MPa respectively,
(Dec 2012) (06 Marks)
5. Design a square key for fixing a gear on a shaft of 25 mm diameter. 15 kW Power at 720 rpm is transmitted from the shaft to the gear. The allowable compressive stress in the material is $150 M P a$ and allowable shear stress is 88 MPa .
6. Design a square key for fixing a gear on a shaft of 25 mm diameter. Shaft is transmitting 15 kW power at 720 rpm to gear. Factor of safety is 3 . For key material yield strength in compression and tension is assumed to be equal and is $460 \mathrm{~N} / \mathrm{mm}^{2}$. Determine dimension of key. (July/June 2013)(10 Marks)
7. A square key $12 \mathrm{~mm} \times 12 \mathrm{~mm}$ is used to transmit a power of 100 kW at 560 rpm. The key is made up of SAE 1045 annealed steel. Using the ASME code procedure, determine the length of the key required.
(Dec. 2010)(05 M)
8. A square key is used to key a gear and a shaft of diameter 35 mm . the hub length of the gear is 60 mm , both key and shaft is made of same material having allowable shear stress of 55 MPa . What are the dimensions of the key according to maximum stress theory if $395 \mathrm{~N}-\mathrm{m}$ of torque is to be transmitted?
(Dec.15/Jan.16) (05 Marks)
9. A $19 \mathrm{~kW}, 1440 \mathrm{rpm}$ motor has a steel shaft, extension of the shaft is 75 mm . diameter of the shaft is 45 mm . maximum torque is 3.5 times the average torque and yield shear stress is 54 MPa , crushing stress is 108 MPa for key material. Design the key and also determine effect of key way. Take factor of safety as 2.5.
(June/July 2016)(Marks 10)
10. Find the length and thickness of a sunk key for a shaft of 100 mm diameter. Assume that the shear resistance of the material of the key is same as that of the shaft. Take the width of the key as 25 mm and the shear stress is equal to 0.4 times the crushing stress.
(Dec.08/Jan.09) (08 Marks)
11. A 45 mm diameter, shaft is made of steel with a yield strength of $40 \mathrm{MPa} . \mathrm{A}$ parallel key size 14 mm wide and 9 mm thick, made of steel with a yield strength of 340 MPa . Find the required length of key, if the shaft in loaded to transmit the maximum permissible torque. Design based on maximum shear stress theory and the factor of safety as 2 .

## KEYS

The most common function of a key is to prevent relative rotation between a shaft and the parts mounted on it such as the hub of a gear, pulley, etc. a key is a piece of metal fitted into mating grooves in the shaft and the hub. The groove of the shaft in which the key lies is called key seat and the groove of the hub in which the key accommodates or fit is called keyway. Thus after assembly the key is partly in the shaft and partly in the hub of the mating part as shown in fig.

An extensive use of key joints is largely due to their simple and reliable design, convenience of assembly and disassembly, low cost, etc.

The major disadvantages of this type of joint are as follows:

1. The need for a keyway makes the effective cross section of the part smaller and increases stress concentration.
2. Difficulty in concentric fitting of parts.

Keys are generally made of mild steel as it can easily take up crushing and shearing stresses to which they are usually subjected. A large number of keys are available and the choices in any installation depends on several factors, such as power requirement, tightness of fit, stability of connection and cost.

## Type of keys

The type of keys in general engineering use are listed below:

1. Saddle key,
2. Flat key,3. Sunk key,
3. Pin key, 5. Taper key.

## Sunk key

A key which goes partly in the shaft and partly in the hub is called sunk key. There are four types of sunk keys.
i). Rectangular or square key,
ii). Gib - head key,
iii). Feather key,
iv). Woodruff key.

## Strength of square and rectangular keys

When the keyways are cut in both the shaft and the hub, torque is transmitted by compression on the surface ab and de of the keys as shown in Fig. these compressive forces acts as a couple tending to shear and the key across eb or crush the key along ab or de.


Let $\mathrm{T}=$ Torque transmitted by the key, $\mathrm{d}=$ Diameter of the shaft, $L=$ Length of the key
$\mathrm{b}=$ Width of the key, $\mathrm{h}=$ Height of the key, $\tau_{1}=$ Allowable shear stress in the key, $\sigma_{b 1}=$ Allowable crushing stress in the key.
Force acting on the circumference of the shaft, $\quad F=\frac{2 T}{d}$
Shear area of the key,

$$
\begin{gathered}
A_{s}=b L \\
\tau_{1}=\frac{F}{A_{s}}=\frac{2 T}{d} \times \frac{1}{b L}
\end{gathered}
$$

Shear stress in the key,

$$
T=\frac{1}{2} \tau_{1} b L d \quad \boldsymbol{E}(\mathbf{4} .6)(\boldsymbol{P}-\mathbf{5 4})
$$

Crushing area,

$$
A_{c}=\frac{h}{2} \times L=\frac{h L}{2}
$$

Crushing (Bearing) stress in the key,

$$
\sigma_{b 1}=\frac{F}{A_{c}}=\frac{2 T}{d} \times \frac{2}{h L}=\frac{4 T}{d h L}
$$

$$
T=\frac{1}{4} \sigma_{b 1} h d L
$$

$$
E(4.5 b)(P-53)
$$

The keyway cut in the shaft affects the load carrying capacity of the shaft since highly localized stresses occurs at and near the corner of the keyway. The code for transmission shafting recommends the use of an efficiency of $75 \%$ for keyed shafts.
$\therefore$ Torque transmitted by the shaft,

$$
\mathrm{T}=\frac{\pi \mathrm{d}^{3} \eta \tau_{1}}{16}
$$

Where $\eta$ is the keyway factor which is equal to 0.75 .

## 12. Prove that a square key is equally strong in crushing and in shear. <br> (May 2017)(04 M)

Solution: Torque transmitted by the key considering shear resistance,

$$
\mathrm{T}_{\mathrm{s}}=\frac{1}{2} \tau_{1} \mathrm{bLd} \quad \mathbf{E}(\mathbf{4} .6) \mathbf{P}-\mathbf{5 4}
$$

Torque transmitted by the key considering crushing resistance,

$$
\begin{gathered}
\mathrm{T}_{\mathrm{c}}=\frac{1}{4} \sigma_{\mathrm{b} 1} \mathrm{hdL} \quad \mathbf{E}(\mathbf{4 . 5 b}) \mathbf{P}-\mathbf{5 3} \\
\frac{\mathrm{T}_{\mathrm{s}}}{\mathrm{~T}_{\mathrm{c}}}=\frac{\frac{1}{2} \tau_{1} \mathrm{bL} \mathrm{~d}}{\frac{1}{4} \sigma_{\mathrm{b} 1} \mathrm{~h} \mathrm{dL}}
\end{gathered}
$$

For square key, $\mathrm{b}=\mathrm{h}$
$\frac{\mathrm{T}_{\mathrm{s}}}{\mathrm{T}_{\mathrm{c}}}=\frac{2 \tau_{1}}{\sigma_{\mathrm{b} 1}}=1 \quad \because$ For ductile material, $\sigma_{\mathrm{b} 1}=\frac{\tau_{1}}{2}$

$$
\text { i.e., } \quad T_{s}=T_{c}
$$

$\therefore$ The key is equally strong in shear and crushing.

## Fasteners

Fasteners can be classified as permanent or detachable fasteners. In case of permanent fasteners, the member cannot be disabled without destruction of joint member or medium.


## KNUCKLE JOINT

## NUMERICAL PROBLEMS:

1. Design and sketch the assembly of a knuckle joint, to connect two mild steel rods, subjected to an axial pull of 100 kN . The allowable stresses for rods and pin are $100 \mathrm{MPa}, 130 \mathrm{MPa}$ and 60 MPa in tension, crushing and shear, respectively. (IP)(Dec 2010) (AU) (Dec 06/Jan 07(Marks 10))
2. Design and sketch the assembly of a knuckle joint, to connect two mild steel rods, subjected to an axial pull of 100 kN . The allowable stresses for rods and pin are $100 M P a, 130 M P a$ and $60 M P a$ in tension, crushing and shear, respectively. the bending of the pin is prevented by selection of proper fit.
(Dec 14/Jan 15)(Marks 08)
3. Design a knuckle joint to transmit load of 120 kN . The design stresses may be taken as 70 MPa in tension, 60 MPa in shear and 150 MPa in compression.
(Dec 16/Jan 17) (IP) (June / July 2011) (Marks 08)
4. Design a knuckle joint to transmit load of 100 kN . The design stresses may be taken as 70 MPa in tension, 60 MPa in shear and 150 MPa in compression.
(AU) (Dec 06/Jan 07) (IP) (Dec 2011)(Marks 10)
5. Design a knuckle joint to transmit load of 150 kN . The design stresses may be taken as 75 MPa in tension, 60 MPa in shear and 150 MPa in compression. (IP) (June 2012)(AU) (July 2007) (ME) (June / July 2011) (Marks 10)
6. Design a knuckle joint suitable for connecting two rods subjected to axial force of 12 kN . The permissible stresses are 40 MPa in tension, 80 MPa in compression and 32 MPa in shear. Give a neat dimensioned sketch.
(Dec.13/Jan.14) (Marks 10)
7. Design a knuckle joint to transmit load of 150 kN . The design stresses may be taken as 80 MPa in tension, 40 MPa in shear and 120 MPa in crushing.
(ME) (Dec 09/Jan 10) (Marks 10)
8. Two mild steel rods, circular in cross section, are required to be connected permitting angular movement. Select a suitable joint and design the same for a maximum load of 35000 N . the permissible stresses for all the components in the joint are:
i. 70 MPa in tension and compression,
ii. 45 MPa in shear,
iii. $40 M P a$ in bearing.
(AU) (June / July 2008) (Marks 10)
9. Design a knuckle joint for the tie rod of circular cross section to sustain a maximum tensile load of 70 kN . The ultimate strength of tie rod against tension is $420 \mathrm{~N} / \mathrm{mm}^{2}$. The ultimate tensile and shearing stresses for the pin material are $500 \mathrm{~N} / \mathrm{mm}^{2}$ and $360 \mathrm{~N} / \mathrm{mm}^{2}$ respectively. Take a factor of safety of 6 .
(ME)(Dec 2011)(Marks 14)
10. Design a knuckle joint to connect two mild steel rods to sustain an axial pull of 150 kN . The pin and rods are made of same material. Assume the working stress in the material as 80 MPa in tension, 40 MPa in shear and 120 MPa in cursing.
(ME) (Dec 09/Jan 10) (Marks 10)

## Knuckle joint

These joints are used to connect two rods that are subjected to tensile loads. If the rods are properly guided, they can support compressive loads also. Where as a knuckle joint form a rigid connection, this joint may allow limited relative angular movement of the rods about the axis of the pin. This joint can be readily disconnected for adjustments or repairs.

## Knuckle joints find applications in

i. Connecting tensions rods of structures.
ii. Rods of valve gears, value rods and eccentric rods.
iii. For joining the links of suspension bridges.
iv. For diagonal strays in boilers
v. Links of cycle chain etc.

## Knuckle joint has three parts namely (i) the eye (ii) the fork and (iii) the pin.

One end of the rod is forged to the shape of a fork (on double eye) and the other rod is forged to the shape of an eye. The eye end of the rod is inserted between the jaws of fork. The hole in to fork and the eye are aligned and a pin having a collar is inserted in the holes. A taper pin or a split pin is inserted in the pin. Both the fork eye ends are octagonal in shape for some length and square for the remaining length.

## Nomenclature:

## Yoke end (fork):

$\mathrm{d}=$ diameter of the rod, $\mathrm{mm} . \quad \mathrm{S}=S_{1}=$ width of the square section, mm.
$\mathrm{D}=$ outside diameter of the eye, $\mathrm{mm} . \quad \mathrm{B}=$ thickness of the fork, mm.
Eye end: $\mathrm{F}=$ thickness of the single eye, mm.

## Knuckle pin:

$d_{1}=$ diameter of the pin, mm. $\quad \mathrm{C}=$ diameter of the pin head, mm.
$\mathrm{E}=$ thickness of the pin head, mm.
$\mathrm{P}=$ axial load, N .
$\sigma_{t}=$ permissible tensile stress, $\mathrm{N} / \mathrm{mm}^{2}$.
$\tau=$ allowable shear stress, $\mathrm{N} / \mathrm{mm}^{2} . \quad \sigma_{c}=$ safe crushing stress, $\mathrm{N} / \mathrm{mm}^{2}$.


## Design procedure of knuckle joint

The empirical dimensions and relations should be considered for designing. The following procedure may be adopted:

## 1. Design of rod and end <br> Figure 1.1

i. Considering the failure of rod in tension, tensile stress in the rod, $\quad \sigma_{t}=\frac{P}{\frac{\pi}{4} d^{2}}$ Tearing resistance of the rod,

$$
\mathrm{P}=\frac{\pi}{4} d^{2} \sigma_{t} \quad \boldsymbol{E}(\mathbf{4} .34)(\boldsymbol{P}-\mathbf{5 7})
$$

ii. Diameter of rod end,

$$
S=S_{1}=d_{2}=1.2 d
$$



Fig. 1.1

## 11. Design of knuckle pin

i. Diameter of knuckle pin,

Considering the failure of knuckle pin in double shear,

$$
\tau=\frac{P}{2 \times \frac{\pi}{4} d_{1}{ }^{2}}
$$

Shearing resistance of the knuckle pin,

$$
\begin{aligned}
& \mathrm{P}=\frac{\pi}{4} d_{1}{ }^{2} \times \tau \times 2 \\
& \mathrm{E}(4.36)(\boldsymbol{P}-57)
\end{aligned}
$$


ii. The diameter of the pin head, ' C ' $\mathrm{C}=1.5 \mathrm{~d} \mathrm{~mm}$
iii. Thickness of the pin head, ' $E$ ' $\mathrm{E}=0.5 \mathrm{~d} \mathrm{~mm}$

## 4. Thickness of the fork: (B)

Considering the failure of fork end in crushing,

$$
\begin{gathered}
\sigma_{c}=\frac{P}{2 B d_{1}} \\
P=2 B d_{1} \sigma_{c} \\
\mathbf{E}(\mathbf{4 . 4 2})(\boldsymbol{P}-\mathbf{5 7})
\end{gathered}
$$



## 5. Outside Diameter of the fork eye :(D)

Considering the failure of fork eye in tension,

$$
\sigma_{t}=\frac{P}{2\left(D-d_{1}\right) B}
$$

$$
P=2\left(D-d_{1}\right) B \sigma_{t} \mathbf{E}(\mathbf{4 . 3 9})(\boldsymbol{P}-\mathbf{5 7})
$$



## 6. Thickness of single eye :( $\mathbf{F}$ )

Considering the failure of single eye in tension, $\quad \sigma_{t}=\frac{P}{F\left(D-d_{1}\right)}$

$$
P=F\left(D-d_{1}\right) \sigma_{t} \quad \mathbf{E}(\mathbf{4 . 4 0})(\boldsymbol{P}-\mathbf{5 7})
$$




## 7. Bending stress in pin :( $\left.\sigma_{b}\right)$

Since the bending of the pin is prevented by the selection of proper fit.
8. Check for Stress: $\quad$ Crushing stress in pin, $\sigma_{c}$

$$
\begin{gathered}
P=F d_{1} \sigma_{c} \quad \mathbf{E}(\mathbf{4 . 4 1})(\boldsymbol{P}-\mathbf{5 7}) \\
\sigma_{c}=\frac{P}{F d_{1}}
\end{gathered}
$$



Note: In case the induced stress $\left(\sigma_{c}\right)$ is more than the allowable stress, then the corresponding dimensions may be increased.

## COTTER JOINT:

## NUMERICAL PROBLEMS:

1. Design a cotter joint to join two round rods capable of sustaining an axial load of 100 kN .
(AU) (July 2006)(Marks 10)
2. A cotter joint is to be designed to connect two rods to carry a load of 120 kN . If the socket and cotter are made of SAE 1045 steel, design the cotter joint.
(Dec. 2010)(10 M)
3. Design a socket and spigot type of cotter joint to connect two rods subjected to a steady axial pull of 100 kN . The material used to spigot end, socket end and cotter is $C 40$ steel, take FOS as 4 for tension, 6 for shear and 3 for crushing based on tensile yield strength.
(ME) (June / July 2009)(Marks 10)
4. Design and sketch the assembly of a cotter joint to connect two rods, subjected to an axial pull of 600 kN . The material selected for the joint has the following permissible stresses: $300 M P a$ in tension, $220 M P a$ in shear and $450 M P a$ in crushing.
(ME)(Dec. 2011)(Marks 12)
5. Design a socket and spigot joint for an axial load of 30 kN . Compression and 30 kN . Tension. The material has the following properties:

$$
\sigma_{t}=50 \mathrm{~N} / \mathrm{mm}^{2}, \sigma_{c}=90 \mathrm{~N} / \mathrm{mm}^{2}, \tau=35 \mathrm{~N} / \mathrm{mm}^{2}
$$

(IP)(Dec. 2012)(Marks 15)
6. Design a socket and spigot cotter joint to sustain an axial load of 100 kN . The material selected for the joint has the following design stresses. $\sigma_{t}=$ $100 \mathrm{~N} / \mathrm{mm}^{2}, \sigma_{c}=150 \mathrm{~N} / \mathrm{mm}^{2}, \tau=60 \mathrm{~N} / \mathrm{mm}^{2}$
(ME)(Dec. 2012)( (IP) (Dec.08/Jan.09)(Marks 10)
7. Design a socket and spigot cotter joint to sustain an axial load of 100 kN . Allowable stress in tension 80 MPa . Allowable stress in compression 120 MPa . Allowable shear stress 60 MPa . Allowable bearing pressure 40 MPa .
8. Design a cotter joint to sustain an axial load of 50 kN . The material selected for the joint has the following stress values : $\sigma_{t}=80 \mathrm{~N} / \mathrm{mm}^{2}, \sigma_{c}=$ $60 \mathrm{~N} / \mathrm{mm}^{2}, \tau=60 \mathrm{~N} / \mathrm{mm}^{2}$. Draw the sectional front view of the joint. (June/July 2014) (IP) (May/June 2010)(Marks 20)
9. Design a cotter joint to sustain an axial load of 50 kN . The allowable stresses are 90 MPa in tension, 60 MPa in shear and 150 MPa in crushing.
(June/July 2014) (Marks 10)
10. Design a cotter joint for an axial load of 50 kN which alternately changes from tensile to compression, assuming allowable stresses in the components under tension and compression as 52.5 MPa , bearing stress as 63 MPa and shearing stress as 35 MPa . Sketch neatly the joint and show dimensions.
(Dec.15/Jan.16)(Marks 15)
11. Design a socket and spigot type of cotter joint to connect two rods subjected to a steady axial pull of 100 kN . The material used to spigot end, socket end and cotter is ( 640 steel) having tensile yield strength of 328.6 MPa. Take FOS as 4 for tension, 6 for shear and 3 for crushing based on tensile yield strength.
(ME) (Dec.07/Jan.08)(Marks 12)
12. Design a socket type of cotter joint for a pull or push of 30 kN . Assume that the cotter and the rod are all of the same material having the following permissible stress, $\quad \sigma_{t}=50 \mathrm{~N} / \mathrm{mm}^{2}, \sigma_{c}=70 \mathrm{~N} / \mathrm{mm}^{2}, \tau=$ $40 \mathrm{~N} / \mathrm{mm}^{2}$ for rod and socket material. $\sigma_{b}=50 \mathrm{~N} / \mathrm{mm}^{2}, \tau=40 \mathrm{~N} / \mathrm{mm}^{2}$ For cotter material.
(IP) (June / July 2013)(Marks 08)
13. Design a sleeve type cotter joint, to connect two tie rods, subjected to an axial pull of 60 kN . The allowable stresses of $C 30$ material used for the rods and cotters are $\sigma_{t}=65^{N} / \mathrm{mm}^{2}, \sigma_{c}=75 \mathrm{~N} / \mathrm{mm}^{2}, \tau=35 \mathrm{~N} / \mathrm{mm}^{2}$; cast steel used for the sleeve has the allowable stresses $\sigma_{t}=70 \mathrm{~N} / \mathrm{mm}^{2}, \sigma_{c}=$

$$
110 \mathrm{~N} / \mathrm{mm}^{2}, \tau=45 \mathrm{~N} / \mathrm{mm}^{2} .
$$

(ME) (May/June 2010)(Marks 10)
14. Two rods made up of plain carbon steel having, tensile stress $\sigma_{y}=$ 380 MPa are to be connected by means of cotter joint, diameter of each rod is 50 mm and cotter is made up of steel of 15 mm thickness. Calculate dimensions of the socket end, making following assumptions.
i. Yield strength in compression is twice of yield strength.
ii. Yield strength in shear is $50 \%$ of tensile yield strength. Taking factor of safety as 6 .
(June / July 2016)(Marks 10)

## Cotter joint

One of the most common methods of joining two non - rotating members is by the use of a cotter. A cotter is a flat wedge shaped member of steel, which is used to rigidly connect rods which transmit linear rotation, cotter joint direction of their length without rotation. Cotter joint transmit both tensile as well as compressive stress.


## Application

i). Used for connecting piston rod with cross head of a double acting steam engine.
ii). For cotter foundation bolts
iii). For connecting two halves of flywheel
iv). For the rods of steel structure
v). For jigs and fixtures

## Merits and demerits of cotter

## Merits

i). Simple in design
ii). Easy assembly and disassembly of the unit
iii). The parts occupy exactly the same relative positions after assembly

## Demerits

i). Weakening of the main parts due to the holes for cotters.
ii). Need for locking devices

## Types of cotter joints

The important types of cotter joints are,
i). Socket and spigot cotter joint
ii). Sleeve and cotter joint
iii). Gib and cotter joint

## Socket and spigot cotter joint

This is used to connect two circular rods which are subjected to axial loads. This joint comprises of three parts

1) The rod end or spigot end
2) The socket end or sleeve end
3)A cotter

## Symbols for cotter joint

$d=$ diameter of the solid rod, $\mathrm{mm}, \quad d_{1}=$ Diameter of the rod end, mm $d_{2}=$ Diameter of the collar $, m m, \quad D=$ Diameter of the socket,$m m$ $D_{1}=$ Diameter of the socket across the cotter hole, mm
$b$ or $B=$ Width of the cotter, mm
$t=$ Thickness of the cotter, $\mathrm{mm}, \quad t_{1}=$ Thickness of the spigot collar, mm
$\sigma_{t}=$ allowable tensile stress, $\quad M P a\left(N / \mathrm{mm}^{2}\right)$
$\sigma_{c}=$ allowable crushing stress, $\quad M P a\left(N / m^{2}\right)$
$\sigma_{b}=$ allowable bending stress, $\quad M P a\left(N / m^{2}\right)$
$\tau=$ allowable shear stress, $\quad M P a\left(N / \mathrm{mm}^{2}\right)$
$\tau^{\prime}=$ allowable shear stress parallel with fibre, $\quad M P a\left(N / \mathrm{mm}^{2}\right)$
Design procedure for socket and spigot cotter joint

## 1. Design of rod (Fig. 1.1)

Axial stress in the rod, $\sigma=\frac{4 P}{\pi d^{2}}$

$$
P=\frac{\pi}{4} d^{2} \sigma_{t} \quad \boldsymbol{E} 4.20(\boldsymbol{P}-\mathbf{5 6})
$$



Fig. 1.1

## 2. Design of spigot and cotter

i. Crushing strength of cotter,
(Fig. 1.2 )
$P=d_{1} t \sigma_{c}$
(i) $E 4.22(P-56)$
$\therefore d_{1} t=$ $\qquad$ ....


Fig. 1.2
ii. Since the weakest section of the spigot is across the slot

$$
P=\left(\frac{\pi}{4} d_{1}^{2}-d_{1} t\right) \sigma_{t} \quad \text { E4.21 }(P-56)
$$

$\therefore$ Axial stress across the slot of the rod,

$$
\sigma_{t}=\frac{4 P}{\pi d_{1}^{2}-4 d_{1} t}
$$

$\therefore$ Diameter of spigot, ${ }^{\prime} d_{1}{ }^{\prime}$
substituting in (i), $\mathrm{t}=$ thickness of cotter

iii. Considering double shear of cotter, (Fig.1.3)

Shearing strength of cotter,

$$
\begin{equation*}
P=2 b t \tau \tag{P-56}
\end{equation*}
$$


iv. Shear stress due to double shear at rod end,

$$
\begin{gathered}
\tau^{\prime}=\frac{P}{2 d_{1} l_{1}} \\
P=2 d_{1} l_{1} \tau^{\prime} \quad E \mathbf{4 . 2 6}(\boldsymbol{P}-\mathbf{5 6})
\end{gathered}
$$

$\therefore$ Rod end distance from the slot ${ }^{\prime} l_{1}{ }^{\prime}$


## 3. Design of spigot collar

(Fig. 1.4)
i. Considering the crushing failure of collar,

$$
P=\frac{\pi}{4}\left(d_{2}^{2}-d_{1}^{2}\right) \sigma_{c} E 4.23(P-56)
$$

$\therefore$ Bearing stress in the collar,

$$
\sigma_{c}=\frac{4 P}{\pi\left(d_{2}{ }^{2}-d_{1}{ }^{2}\right)}
$$



Fig. 1.4


Fig. 1.5
$\therefore \quad$ Diameter of spigot collar ${ }^{\prime} d_{2}{ }^{\prime}$
ii. Considering the failure of collar in shear,
(Fig. 1.5)

$$
P=\pi d_{1} t_{1} \tau \quad \boldsymbol{E} 4.24(P-56)
$$

$\therefore$ Shear stress induce in the collar,
$\tau=\frac{P}{\pi d_{1} t_{1}}$
$\therefore$ Thickness of spigot collar ${ }^{\prime} t_{1}{ }^{\prime}$


## 4. Design of socket

i. Considering the failure of the socket in tension or compression,
(Fig. 1.6)
$P=\left[\frac{\pi}{4}\left(D_{1}{ }^{2}-d_{1}{ }^{2}\right)-\left(D_{1}-d_{1}\right) t\right] \sigma_{t} \boldsymbol{E} 4.27(\boldsymbol{P}-56)$
$\therefore$ Tensile stress induce in the socket,

$$
\sigma_{t}=\frac{4 P}{\pi\left(D_{1}{ }^{2}-d_{1}^{2}\right)-4 t\left(D_{1}-d_{1}\right)}
$$

$\therefore$ Outside Diameter of socket ${ }^{\prime} D_{1}{ }^{\prime}$


Fig. 1.6
(Fig. 1.7)
$P=\left(D-d_{1}\right) t \sigma_{c} \quad \boldsymbol{E} 4.28(\boldsymbol{P}-56)$
$\therefore$ Crushing stress induce in the socket collar,

$$
\sigma_{c}=\frac{P}{\left(D-d_{1}\right) t}
$$

$\therefore$ Diameter of socket collar ' $D^{\prime}$


Fig. 1.7
iii. Considering the failure of socket end in double shear,
$\therefore$ Shear stress induce in the socket end, $\quad \tau^{\prime}=\frac{P}{2\left(D-d_{1}\right) l}$
$\therefore$ Thickness of socket collar ' $l^{\prime}$

iv. Considering the failure of socket in shear,

$$
P=\pi d t_{2} \tau^{\prime} \quad \text { E4.31 }(P-56)
$$

$\therefore$ Shear stress induce in the socket, $\tau^{\prime}=\frac{P}{\pi d t_{2}}$
$\therefore$ Thickness of socket at the rod end ${ }^{\prime} t_{2}{ }^{\prime}$

5. Bending stress induced in the collar,(Fig. 1.9)

Maximum bending moment to resisting moment,

$$
P=\frac{4}{3} \frac{t b^{2} \sigma_{b}}{D} \quad \quad \boldsymbol{E} 4.33(P-56)
$$



Fig. 1.9
This bending stress induced in the cotter should be less than the allowable bending stress of cotter. In case induced bending stress is more than the allowable value then ' $t$ ' or ' $b$ ' may be increased.

## MECHANICAL COUPLINGS

Whenever lengths of shafting exceeds some 7 meters in length, it is made up of two or more lengths. In such conditions is becomes necessary to join the ends of two shafts in such a manner that both the shafts acts as the same unit. The elements which join two shafts are known as couplings or clutches.

Couplings are used to connect and to transmit power from the driving shaft to the driven shaft

A Coupling is used to connect two shafts permanently. It is disconnected only for repairs or to make a change in the instantiation. Permanent couplings are referred to simply as couplings, while those which may be rapidly engaged to transmit power, or disengaged when desire, are called clutches.

Couplings are also used to connect driving unit (turbine, electric motor or an engine) and driven unit (pump, compressors etc.)- Couplings are available for joining shafts at angles or with misalignment, and have ability to provide damping.

## SELECTION

The choice of a coupling is based on the following considerations:
(i) Loading: Torque to be transmitted and type of load, i.e., static, variable or shock.
(ii) Misalignment: The maximum parallel and/or angular misalignment or relative position of the shafts to be joined. Requirement of compensation of axial displacement.
(iii) Length: To get the length of shaft.
(iv) Repair: To provide for disconnection for repairs or alterations.
(v) Requirement of damping ability.
*** What is coupling \& what are the requirements of good coupling
The important requirement of good couplings are (ME)(Dec.2012)(Marks 4)
i. It must transmit the full torque of the shaft
ii. It must be keep the shafts in perfect alignment
iii. It must be easy to assemble or dissemble
iv. The bolt heads, nuts and other projecting parts should be protected by suitable flanges; rims or cover plates.

## CLASSIFICATION

A wide variety of couplings are commercially available, ranging from a simple rigid coupling to elaborate flexible couplings using gears, elastomer and fluids for transmission of torque from one shaft to another shaft or from a shaft to a device.
*** What is coupling \& state different types shaft couplings?
(IP) (June / July 2013)(Marks 04)


Couplings may be classified in two broad classes: rigid and flexible couplings.
Rigid Couplings: Rigid couplings are used to connect two shafts which are collinear. They do not permit any relative rotation and axial motion between them. The most widely used rigid couplings are:
(i) Sleeve or Box or Muff coupling.
(ii) Split Muff coupling.
(iii) Marine or Solid flange coupling.
(iv) Cast iron flange coupling.

Flexible coupling: Flexible couplings are used to connect two shafts which are non-collinear i.e., shafts having slight parallel or angular misalignment. A flexible coupling permits relative rotation and variation in the alignment of shafts within certain limits. The most widely used Flexible couplings are:
(i) Pin or bush type flexible coupling
(ii) Oldham's coupling
(iii) Universal coupling
(iv) Band type flexible coupling.
(v) Fabric flexible coupling.

A protective cover is provided on each flange, to ensure that the clothes of a worker do not get entangled with running bolts and nuts.
*** Distinguish between a rigid coupling and a flexible coupling.
(IP) (Dec. 2012) (Marks 05)

| Rigid coupling | Flexible coupling |
| :--- | :--- |
| A rigid coupling cannot tolerate <br> misalignment between the axes of the <br> shafts, it can be used only when there <br> is precise alignment between two <br> shafts. | The flexible coupling, due to <br> provision of flexible elements like <br> bush or disc or pin, can tolerate $0.5^{0}$ <br> of angular misalignment and 5 mm of <br> axial displacement between the shafts. |
| It can be used only where the motion <br> is free from shocks and vibration. | It can be used where the motion is <br> with shocks and vibration. |
| Simple and inexpensive. | Comparatively costlier due to <br> additional parts. |

In practice, misalignment always exists due to imperfect workmanship. Therefore, flexible couplings are more popular.

## RIGID FLANGE COUPLING:

## THEORY QUESTIONS:

i. What is coupling \& what are the requirements of good coupling
(ME)(Dec. 2012)(Marks 04)
ii. What is coupling \& state different types shaft couplings?
(IP) (June / July 2013)(Marks 04)
iii. Distinguish between a rigid coupling and a flexible coupling.
(IP) (Dec. 2012) (Marks 05)

## NUMERICAL PROBLEMS:

1. Design a rigid coupling to transmit 25 kW at 500 rpm . Select suitable materials for shaft, key and bolts. (ME)(Dec. 08/Jan. 09) (Marks 14)
2. Design a CI flange coupling for a steel shaft transmitting 15 kW at 200 rpm . The allowable sheer stresses in shaft, bolt and key materials is 40 MPa and the allowable sheer stress in flange is 20 MPa . The maximum torque is $25 \%$ greater than the mean torque.
(June/July 2011)(Marks 12)
3. Design a cast iron protective flange coupling to transmit 15 kW at 900 rpm from an electric motor to a compressor. The following permissible stresses may be used. The shear stress for shaft and key materials as $40 M P a$. The crushing stress for bolt material is $80 M P a$ and shear stress bolt material is $40 M P a$. The crushing stress for key is $80 M P a$ and shear stress for cast iron is 8 MPa. Assume safety factor is 1.35.` (IP)(June/July 2013) (Marks 08)
4. Design a protected type cast iron flange coupling for a steel shaft transmitting 30 kW at 200 rpm . The allowable shear stress in the shaft and key material is 40 MPa . The maximum torque transmitted to be $20 \%$ greater than full load torque. The allowable shear stress in the bolt is $60 M P a$ and allowable shear stress in the flange is 40 MPa .
(IP)(Dec. 08/Jan.09) ( Marks 12) (AU) (Dec. 06/Jan.07) (Marks 15) (ME)(May / June 2010) ( Marks 10)
5. Design a protected type cast iron flange coupling for a steel shaft transmitting 15 kW at 200 rpm and having an allowable shear stress of 40 MPa . The permissible shear stress in the bolt is 30 MPa . Assume the same material used for shaft and key and that the crushing stress is twice the shear stress. The maximum torque transmitted to be $25 \%$ greater than full load torque. The shear stress for cast iron is 14 MPa .
(IP)(June 2012) (Marks 10)
6. Design a flange coupling to transmit 14 kW at 600 rpm . Select $C 40$ steel for the shaft and C35 steel for bolts, with a factor of safety $=2$. Use allowable shear stress in cast iron flanges as 15 MPa . Also draw the sketch of the coupling.
(IP)(Dec. 2010)(Marks 10)
7. It is required to design a rigid type flange coupling to connect two shaft transmits 37.5 kW at $180 \mathrm{rev} / \mathrm{min}$ to the output shaft through the coupling. The starting torque is $50 \%$ higher than the rated torque. Select material for flanges as cast iron $F G 200\left(\sigma_{u t}=200 M P a\right)$ with a factor of safety 6 , material for shafts as carbon steel with $\sigma_{y t}=380 M P a$, with a factor of safety 2.5, material for key and bolts may be taken as steel with $\sigma_{y t}=400 M P a,($ in tension $)$ and $\sigma_{y c}=600 M P a($ in compression $)$ respectively and a factor of safety 2.5 . Design the coupling and give major dimensions.
(ME)(June 2012) (Marks 14)
8. a flange coupling connects two shafts of 50 mm diameter of commercial shafting. The flange coupling is bolted together, with 4 bolts of the same material as the shaft. The diameter of the bolt circle is 250 mm and the web thickness is 22 mm .
i. Determine the minimum hole diameter to transmit the same torque as that of the shaft. Consider both shear strength and bearing strength in bolts.
ii. What is the power transmitted at 200 rpm ? Assume an allowable shear stress 40 MPa .
(Dec. 2011)(10 M)
9. A mild steel has to transmit 75 kW at 200 rpm . The allowable stress in the shaft material is limited to 40 MPa and the angle of twist is not to exceed $1^{0}$ in a length of 20 diameters. Calculate the suitable diameter of the shaft and also design a cast iron flange coupling for this shaft. Assume the allowable stress in the material of the bolts to be $30 M P a$ and the bolts are fitted in reamed holes. Assume the allowable shear stress in the cast iron flange equal to 15 MPa . Assume $G=80 \times 10^{3} \mathrm{~N} / \mathrm{mm}^{2}$.

## (Dec. 16/Jan.17) ((IP)Dec. 09/Jan.10) ((ME)June/July 2008) (Marks 15)

10. A rigid coupling is used to transmit 50 kW power at 300 rpm there are six bolts; the outer diameter of the bolt is 200 mm , while recess diameter is 150 mm the coefficient of friction between flange is 0.15 . The bolts are made up of $45 C 8 \sigma_{y t}=380 \mathrm{~N} / \mathrm{mm}^{2}$ and FOS is 3. (IP)(June/July 2013) (M 10)
11. Design a flanged coupling to connect the shafts of motor and pump transmitting 15 kW power at 600 rpm . Selesct C40 steel for shaft and C35 steel for bolts, with factor of safety $=2$. Use allowable shear for Cast-Iron flanges $=$ $15 \mathrm{MPa}, \quad \sigma_{\text {allowable }}=162 \mathrm{MPa}, \tau_{\text {allowable }}=81 \mathrm{MPa}$ and $\quad$ for bolts $\sigma_{\text {allowable }}=152$ MPa, $\tau_{\text {allowable }}=76$ MPa. $\quad($ June/July 2014)
12. Design a C.I. flange coupling to transmit 18 kW at 1440 rpm . The allowable stresses for shaft, keys and bolts are 75 MPa in shear and 150 MPa in crushing. The allowable shear stress for C.I. flange is 5 MPa .
( June/July 2014) (Marks 10)
13. Design a rigid flange coupling to transmit 18 kW at 1440 rpm . The allowable shear stress in the cast iron flange is $4 M P a$. The shaft and the key are made of C40 steel. (ME)(Dec. 16/Jan.17) (Marks 10)
14. Design a rigid flange coupling to transmit 18 kW at 1440 rpm . The allowable shear stress in the cast iron flange is $4 M P a$. The shaft and the key are made of AISI 1040 annealed steel material of allowable shear stress 93.384 MPa. (ME)(Dec. 07/Jan.08) (Marks 20)
15. Design a rigid flange coupling to transmit 18 kW at 1440 rpm . The allowable shear stress in the cast iron flange is $4 M P a$. The shaft and the key are made of AISI 1040 annealed steel material with ultimate strength and yield stress value as 518.8 MPa and 353.4 MPa , respectively. Use ASME code to design the shaft and the key. (ME)( June/July 2013) (M14)
16. A rigid coupling has four bolts on a pitch circle of 125 mm diameter and is transmitting 20 kW power at 720 rpm . The bolts are made of carbon steel (C45) and has the factor of safety 3. Determine the diameter of the bolt. (ME)(Dec. 08/Jan.09) (Marks 06)
17. A cast iron protective type flange coupling is used to connect two shafts of 80 mm diameter. The shaft runs at 250 rpm and transmits a torque of $4300 \mathrm{~N}-\mathrm{m}$. The permissible shear stress for shaft and bolt materials is $50 M P a$ and permissible shear stress for flange is $8 M P a$. Design the bolts, hub and flange for the coupling.
(ME)(Dec. 2011) (Marks 08)
18. In a rigid flange coupling designed to transmit 50 kW at 200 rpm , a tapered key of dimensions $18 \times 11 \times 100$ is used to key the shafts of 60 mm diameter to flange. Five bolts are used on a bolt circle 170 mm diameter. Taking the material of bolts same as that of shaft, determine
i. The shear stress induced in the shaft and key
ii. The size of the bolts required.
(AU)(June/July 2009) (Marks 12)
19. Design a cast iron flange coupling to connect two shafts of 45 mm diameter is to transmit 20 kW power a 400 rpm . The permissible shear strength for the shaft, bolt and key is 50 MPa and the permissible compressive stress is 120 MPa . The permissible shear strength for cast iron is 15 MPa . Assume starting torque is 30 percent higher than the nominal torque. Design the coupling assuming the bolts are fitted in reamed holes.
(Dec. 14/Jan. 15) (Marks 12)
20. Design a flange coupling to connect thee shaft of a motor and centrifugal pump for the following specifications:

Pump output $=3000$ ltrs $/ \mathrm{mm}$; Total head $=20 \mathrm{~m} ;$ Pump speed $=600 \mathrm{rpm} ;$ Pump efficiency $70 \%$; Select C40 steel $\left(\sigma_{y}=328.6 \mathrm{MPa}\right)$ for shaft, bolts and keys with factor of saffety2. Use allowable shear stress in cast iron flanges equal to 15 MPa .
(May 2017) (Marks 10)

Cast Iron Flange Coupling (Protected and Un - Protected CI flange coupling)
It consists of two similar cast iron flanges. Each flange is mounted on the shaft end and keyed to shaft. Sunk keys of rectangular or square cross - section are commonly used for the purpose. The flanges are connected by means of bolts and nuts as shown in Figs (13.2) and ( $\mathbf{1 3 . 3}$ ) ( $\mathbf{P}-\mathbf{2 3 3}$ ). one of the flanges has a projected portion and the other flange has a corresponding recess. To ensure correct alignment, one of the shaft is extended so that its end partially enter the flange of the other shaft. this helps to bring the shafts to be in line and maintain alignment.

In un-protected CI flange coupling as shown in Figs (13.2) (P-233). The bolt heads and nuts are open and hence liable to cause injury to the operator.

In protected CI flange coupling the bolt heads and nuts are covered by the projecting flanges as shown in Figs (13.3) (P- 233)

## Design procedure for (protected / unprotected) cast iron rigid flange coupling

The component to be designed in this coupling are (i). Shafts (ii). Bolts (iii). Key (iv). Hub and (v) Flange.

Note : Refer Fig (13.2)(P-233)

## i). Shaft design

Torque transmitted by the coupling

$$
T=\frac{9.55 \times 10^{6}(P)}{n} \quad N-m m \quad \boldsymbol{E}(\mathbf{3 . 3} \boldsymbol{a}) \boldsymbol{P} 42
$$

Power, P is in kW
And is equated to the equation for torque transmitted by the shaft diameter given by
Based on strength,

$$
\begin{equation*}
\tau=\frac{16 T}{\pi \eta D^{3}} \tag{3.1}
\end{equation*}
$$

And based on rigidity (stiffness)

$$
\begin{equation*}
\theta=\frac{584 T L}{G D^{4}} \tag{3.2}
\end{equation*}
$$

Where $\mathrm{D}=$ Shaft diameter.
The value of ' $D$ ' is round off to the next standard size Ref Table T(3.5a) P-48

## ii). Key design

Based on the shaft diameter ' $D$ ' the value of width ' $b$ ' and thickness ' $h$ ' of taper key are selected from Table $\mathbf{T}(\mathbf{4 . 2}) \mathbf{P}-\mathbf{6 1}$

Calculate the effective length of key $=\mathrm{L}=$ Hub length

$$
\mathrm{L}=1.2 \mathrm{D}+20 \mathrm{~mm} \quad \boldsymbol{E}(\mathbf{1 3 . 6}) \boldsymbol{P} 210
$$

After selecting $b, h$ and $L$ for the key. Give a check for the induced shear stress and induced crushing stress in the key,

## Check for key

For Induced crushing stress in key
$T=\frac{1}{4} \sigma_{b 1} h d L$
$\boldsymbol{E}(4.5$ b)P 53

$$
\sigma_{k_{(\text {ind })}}=\frac{4 T}{h d L} \quad M P a
$$

For Induced shear stress in key

$$
T=\frac{1}{2} \tau_{1} b d L
$$

$E(4.6) P 54$

$$
\tau_{k_{(i n d)}}=\frac{2 T}{b d L} \quad M P a
$$

$$
\text { if } \begin{gathered}
\tau_{k_{(\text {ind })}}<\tau_{k_{(\text {allow })}}, \quad \text { Design is Safe. } \\
\sigma_{k_{(\text {ind })}}<\sigma_{k_{(\text {allow })},}, \quad \text { DesignisSafe } .
\end{gathered}
$$

If the induced shear stress $\tau_{k_{(\text {ind })}}$ and crushing stress $\sigma_{k_{(\text {ind })}}$ in the key are less than this allowable (permissible) shear stress $\tau_{k_{(\text {allow })}}$ and allowable (permissible) crushing stress $\sigma_{k_{(a l l o w)},}$, the design of key is safe. Otherwise increase the key dimensions and check once again.

## iii). Hub design

The hub diameter

$$
D_{1}=1.8 D+20 \mathrm{~mm}
$$

$$
E(13.2) P 209
$$

The hub length

$$
\mathrm{L}=1.2 \mathrm{D}+20 \mathrm{~mm}
$$

$$
E(13.6) P 210
$$

iv). Bolt design

Number of bolts $\quad i=\frac{1}{40} D+2 \quad$ to $\quad \frac{3}{80} D+2$
$E(13.1) P 209$
Even no of bolts as preferred where ' $D$ ' is diameter of shaft.
Determine the minor diameter of the bolt ' d ' by the empirical formula

$$
\begin{equation*}
d=\frac{0.423 D}{\sqrt{i}}+7.5 \mathrm{~mm} \tag{13.5}
\end{equation*}
$$

Select the size of bolt based on the core area of bolt $\left(\mathrm{A}_{\mathrm{C}}\right)$
Standardize the bolt size using table T(9.8) P-113
Note: unless otherwise specified the shaft, key and bolts are made of same material and hence have same value of stress

$$
T_{\text {Shaft }}=T_{\text {Key }}=T_{\text {Bolt }} \quad \text { i.e., } \tau_{\text {Shaft }}=\tau_{\text {Key }}=\tau_{\text {Bolt }}
$$

Bolt circle diameter

$$
D_{2}=D_{1}+3.2 d \mathrm{~mm}
$$

$$
E(13.3) P 210
$$

## Check for bolt

Check for induced shear stress in bolt, $T=\frac{\tau_{b_{(\text {ind })}} \pi i d^{2} D_{2}}{8} \quad \boldsymbol{E}(\mathbf{1 3 . 9}) \boldsymbol{P} 210$

$$
\begin{gathered}
\tau_{b_{(\text {ind })}}=\frac{T \times 8}{\pi i d^{2} D_{2}} \\
\text { if } \quad \tau_{b_{(\text {ind })}}<\tau_{b_{(\text {allow })}, \quad \text { DesignisSafe. }}
\end{gathered}
$$

If the induced shear stress $\tau_{b_{(\text {ind })}}$ in the bolt are less than this allowable (permissible) shear stress $\tau_{b_{(a l l o w)}}$, the design of key is safe. Otherwise increase the bolt dimensions and check once again.

## v). Flange design

Outer diameter of flange,

$$
D_{3}=D_{1}+6 d m m \quad \boldsymbol{E}(\mathbf{1 3 . 4}) \boldsymbol{P} 209
$$

The flange thickness,

$$
\begin{equation*}
t=0.35 D+9 \mathrm{~mm} \tag{13.7}
\end{equation*}
$$

## Check for flange

$$
\begin{aligned}
& T=\frac{\tau_{f_{(\text {ind })}} \pi{D_{1}}^{2} t}{2} \quad \boldsymbol{E ( 1 3 . 1 1 ) P} 210 \\
& \text { if } \quad \tau_{f_{(\text {ind })}}=\frac{2 \times T}{\pi{D_{1}}^{2} t} \\
& \tau_{f_{(\text {ind })}}<\tau_{f_{(\text {allow })},} \quad \text { Design is Safe. }
\end{aligned}
$$

If the induced shear stress $\tau_{f_{(\text {ind })}}$ in the flange are less than this allowable (permissible) shear stress $\tau_{f_{(\text {allow })}}$, the design of key is safe. Otherwise increase the flange dimensions and check once again.

| Symbols (Un Protected \& protected) |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | Formulae | Equations | Table |
| Design of Shaft |  |  |  |  |


| T | Torque <br> transmitted in $\mathrm{N}-$ <br> mm | $\frac{9.55 \times 10^{6}(P)}{n}$ <br> P in kW | (3.3a) P-42 |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $\frac{16 T}{\pi \eta D^{3}}$ | $\mathbf{( 3 . 1 ) P - 4 8}$ |  |  |
|  |  | $\theta=\frac{584 T L}{G D^{4}}$ | $\mathbf{( 3 . 2 ) P - 4 8}$ | $\mathbf{( 3 . 5 a ) P - 4 8}$ |

Design of Key

| b | Width of taper <br> key |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| h | Height of taper <br> key |  | (4.2) P-61 |  |
| L | Length of taper <br> key $\approx$ Length of <br> Hub | $\mathrm{L}=1.2 \mathrm{D}+20 \mathrm{~mm}$ | (13.6) P-210 | $\mathbf{( 4 . 2 ) ~ P - 6 1 ~}$ |

Check for key

| $\sigma_{k_{(\text {induced }}}$ | Induced crushing <br> stress in key | $T=\frac{1}{4} \sigma_{k_{(\text {ind })} h d L}$ | $\mathbf{( 4 . 5 b )} \mathbf{P - 5 3}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $\tau_{k_{\text {(induced }}}$ | Induced shear <br> stress in key | $T=\frac{1}{2} \tau_{k_{(\text {ind })} b D L}$ | $\mathbf{( 4 . 6 ) ~ P - 5 4}$ |  |

Design of Hub

| $D_{1}$ | hub diameter | $D_{1}$ <br> $=1.8 D+20 \mathrm{~mm}$ | (13.2) P-209 |
| :--- | :--- | :--- | :--- | :--- |$\quad$.

Design of Bolt
$\left.\begin{array}{|l|l|c|c|l|}\hline \begin{array}{l}\text { No of } \\ \text { bolts } \\ \text { pins }\end{array} & / & \mathrm{i} & \begin{array}{l}i=\frac{1}{40} D+2 \\ \text { to } \frac{3}{80} D+2\end{array} & \text { (13.1) P-209 }\end{array}\right]$

Check for bolt

| $\tau_{b_{\text {(induced }}}$ | Induced <br> stress in bolt | $T$ |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $=\frac{\tau_{b_{\text {(ind) }}} \pi i d^{2} D_{2}}{8}$ | (13.9) P-210 |  |  |

Design of flange

| $D_{3}$ | Outer diameter <br> of flange | $D_{3}$ <br> $=D_{1}+6 d \mathrm{~mm}$ | (13.4) P-209 |
| :--- | :--- | :--- | :--- | :--- |$\quad$| t | Flange thickness |
| :--- | :--- |
| $t$ <br> $=0.35 D+9 \mathrm{~mm}$ | $\mathbf{( 1 3 . 7 )} \mathbf{P - 2 1 0}$ |

Check for flange

| $\tau_{f_{(\text {induced })}}$ | Induced shear <br> stress in Flange | $T$ <br> $=\frac{\tau_{f_{\text {(ind) }}} \pi D_{1}{ }^{2} t}{2}$ | $\mathbf{( 1 3 . 1 1 )}$ <br> $\mathbf{2 1 0}$ | $\mathbf{P -}$ |
| :---: | :--- | :--- | :--- | :--- |
| $t_{1}$ | Thickness of <br> protective <br> circumferential <br> flange | Assume <br> $t_{1}=0.25 \mathrm{D} \mathrm{mm}$ |  |  |
| $\eta$ | Key way factor | Assume <br> $\eta=0.75 \mathrm{~mm}$ |  |  |

## FLEXIBLE COUPLING (BUSH-PIN TYPE)

## NUMERICAL PROBLEMS:

1. Design a bush pin type flexible coupling to transmit 25 kW at 500 rpm . Select suitable materials for shaft, key and bolts.
(ME)(Dec. 08/Jan. 09) (Marks 14)
2. A flexible coupling is used to transmit 15 kW power at 100 rpm . Choose the appropriate material for the shaft and the flanges. Also design other components and calculate stresses induced in each component of the coupling.
(Dec. 2010)(15 M)
3. Design a flanged coupling, to transmit a power of 32 kW at 960 rpm . The overall torque is $20 \%$ greater than the mean torque. The allowable shear stress for the shaft, key and bolt is 40 MPa . The allowable shear stress in the CI flange is 15 MPa . Bearing pressure in the bush is 0.8 MPa .
(ME) (Dec. 2011) (Marks 12)
4. A flexible flange coupling has to transmit a power of 45 kW at a rated speed of 500 rpm . Select $C 40$ steel for the shaft and the pin. Assume factor of safety 2.5 . Determine the maximum normal stress, maximum shear stress induced in the pin, torque capacity based on the shearing of bolts.
(ME) (July. 2007) (Marks 08)
5. Design a bush pin type flexible coupling to connect a motor shaft to a pump shaft transmitting 20 kW power at 1440 rpm . The allowable shear and crushing stress for steel shafts, key and pins are $40 M P a$ and $80 M P a$ respectively. The allowable shear stress for the cast iron flange is $10 M P a$ and the allowable bearing pressure for rubber bush is 0.5 MPa .
(ME)(Dec. 09/Jan. 10) (Marks 10)
6. Design a flanged coupling, to transmit a power of 32 kW at 960 rpm . The overall torque is $20 \%$ greater than the mean torque. The allowable shear stress for the shaft, key and bolt is 40 MPa . The allowable shear stress in the CI flange is 15 MPa . Bearing pressure in the bush is 0.8 MPa .

## (Dec. 2011) (Marks 12)

7. Design a flexible bushed pin type coupling suitable for transmitting 40 kW of power at 1000 rpm . The overall torque is 20 percent greater than the mean torque. The material properties are as follows:
i. The allowable shear and crushing stress foe shaft and key material is 40 MPa and 120 MPa respectively.
ii. The allowable shear stress for cast iron is 10 MPa .
iii. The allowable bearing pressure for rubber bush is 0.45 MPa .
iv. The material of the pin is same as that of shaft and key having allowable stress in bending of 152 MPa .

Motor shaft diameter is 50 mm and pump shaft diameter is 45 mm .
(ME Dec. 13 / Jan. 14) (Marks 12)

## FLEXIBLE COUPLING (BUSH-PIN TYPE)

The perfect alignment is rare to achieve in two shafts, The axial, angular and parallel are the most common misalignments that exist either independently or in combination. The misalignments are shown in Figure 13. In such cases the flexible couplings are used and also absorb impact loads due to fluctuations in Torque and speed. The flexibly in a coupling can also be achieved due to the presence of some member itself and it named as incorporated flexibility. Pin or bush type coupling is the common example of flexible coupling.

parallel misalignment, $\delta$

## Bush type flexible coupling

This coupling is a modification of a rigid-type flange coupling in the sense that the design of the shaft and the hub of flanges are similar to that of rigid flange coupling (see Fig. 13.4c)(P-233 A). In this, rubber bushes are provided to act as flexible elements, and to allow for a minor angular misalignment between the two shafts.


A brass sleeve of 2-3 mm thickness is used to reduce wear and tear of the rubber bush with pin. The rubber ( 6 to 10 mm thickness) with brass sleeve ( 3 to 5 mm thickness) is used over the pin and provides the desired flexibility in the Coupling

The flanges of the Coupling are not identical as shown in the figure. No socket and spigot are provided on the two flanges. Rather, there is a clearance of about 5 mm between the two faces of flanges. The designs of the two flanges are different because one flange contains holes for pins and bushes, and the other flange contains holes for the threaded portion of the pin and the nut.

The diameter of the pin is also increased to increase the bearing area (bearing capacity) between bush and the flange hole. It should also be notice that the thickness of the flange containing bush is more to compensate the larger diameter of the hole made for bush. The pitch circle diameter of the pin is also to be on larger side to reduce the bearing pressure between bush and flange. the increased flexibility at the bush induce the possibility of bending of the pin; hence. the pin for bush type flexible coupling should also be designed for bending consideration. The driving flange carries the larger portion of the pin with rubber bush and brass sleeve. A protective cover is provided on this flange to ensure safety. The flanges are made of CI. The shaft, key, bolts/Pins are made of steel.

## Design procedure for (bush pin) cast iron flexible flange coupling

The components to be designed in this coupling are (i). Shafts (ii). Bolts (iii). Key (iv). Hub (v) Flange and (vi) Bush.

Note : Refer Fig (13.4 c)P-233 A

## i). Shaft design

Torque transmitted by the coupling

$$
T=\frac{9.55 \times 10^{6}(P)}{n} \quad N-m m \quad \boldsymbol{E}(\mathbf{3 . 3} \boldsymbol{a}) \boldsymbol{P} 42
$$

Power, P is in kW
and is equated to the equation for torque transmitted by the shaft diameter given by

Based on strength,

$$
\begin{equation*}
\tau=\frac{16 T}{\pi \eta D^{3}} \tag{3.1}
\end{equation*}
$$

And based on rigidity (stiffness),

$$
\begin{equation*}
\theta=\frac{584 T L}{G \eta D^{4}} \tag{3.2}
\end{equation*}
$$

Where $\mathrm{D}=$ Shaft diameter in mm. $\theta$ in Degree
The value of ' $D$ ' is round off to the next standard size Ref Table T(3.5a) P-48

## ii). Key design

Based on the shaft diameter ' $D$ ' the value of width ' $b$ ' and thickness ' $h$ ' of taper key are selected from Table T(4.2) P-61

Calculate the effective length of key $=\mathrm{L}=$ Hub length

$$
\mathrm{L}=1.2 \mathrm{D}+20 \mathrm{~mm} \quad \boldsymbol{E}(\mathbf{1 3 . 6}) \boldsymbol{P} 210
$$

After selecting $b, h$ and $L$ for the key. Give a check for the induced shear stress and induced crushing stress in the key,

## Check for key

For Induced crushing stress in key, $\quad T=\frac{1}{4} \sigma_{b 1} h d L \quad \boldsymbol{E}(\mathbf{4 . 5} \boldsymbol{b}) \boldsymbol{P} 53$

$$
\sigma_{k_{(i n d)}}=\frac{4 T}{h d L} \quad M P a
$$

For Induced shear stress in key $\quad T=\frac{1}{2} \tau_{1} b d L \quad \boldsymbol{E}(4.6) \boldsymbol{P} 54$

$$
\begin{gathered}
\tau_{k_{(\text {ind })}}=\frac{2 T}{b d L} \quad M P a \\
\text { if } \quad \tau_{k_{(\text {ind })}}<\tau_{k_{(\text {allow })}}, \quad \text { Design isSafe. } \\
\sigma_{k_{(\text {ind })}}<\sigma_{k_{(\text {allow })}, \quad \text { DesignisSafe. }}
\end{gathered}
$$

If the induced shear stress $\tau_{k_{(\text {ind })}}$ and crushing stress $\sigma_{k_{(\text {ind })}}$ in the key are less than this allowable (permissible) shear stress $\tau_{k_{(\text {allow })}}$ and allowable (permissible) crushing stress $\sigma_{k_{(\text {allow) }},}$, the design of key is safe. Otherwise increase the key dimensions and check once again.

## iii). Hub design

| The hub diameter | $D_{1}=1.8 D+20 \mathrm{~mm}$ | $\boldsymbol{E}(\mathbf{1 3 . 2}) \boldsymbol{P} 209$ |
| :--- | :--- | :--- |
| The hub length | $L=1.2 D+20 \mathrm{~mm}$ | $\boldsymbol{E}(\mathbf{1 3 . 6}) \boldsymbol{P} 210$ |

iv). Pin / Bolt design

Number of bolts, $\quad i=\frac{1}{40} D+2 \quad$ to $\quad \frac{3}{80} D+2 \quad \boldsymbol{E}(\mathbf{1 3 . 1}) \boldsymbol{P} 209$
Even no of pins/ bolts as preferred where ' D ' is diameter of shaft.
Determine the minor diameter of the pins/ bolts ' $d$ ' by the empirical formula

$$
\begin{equation*}
d=\frac{0.423 D}{\sqrt{i}}+7.5 \mathrm{~mm} \tag{13.5}
\end{equation*}
$$

Select the size of bolt based on the core area of pins/ bolts $\left(\mathrm{A}_{\mathrm{C}}\right)$
Standardize the pins/ bolts size using table T(9.8) P-113

Note: unless otherwise specified the shaft, key and pins/ bolts are made of same material and hence have same value of stress

$$
\begin{gathered}
T_{\text {Shaft }}=T_{\text {Key }}=T_{\text {Bolt }} \\
\text { i.e., } \tau_{\text {Shaft }}=\tau_{\text {Key }}=\tau_{\text {Bolt }}
\end{gathered}
$$

Calculate the major diameter of the pin by empirical relation
$d_{P}=2 \times$ Preliminary bolt diameter of the thread portion

$$
d_{P}=2 \times d
$$

Assume the thickness of the brass bush $t_{b} \& t_{r}$ the rubber bush
Thickness of the brass bush

$$
t_{b}=1 \text { to } 3 \mathrm{~mm}
$$

Thickness of the rubber bush

$$
t_{r}=5 \text { to } 10 \mathrm{~mm}
$$

Inner diameter of the rubber bush $\quad d_{P i}=d_{P}+2 t_{b}$
Outer diameter of the rubber bush $\quad d^{\prime}=d_{P i}+2 t_{r}$
Bolt circle diameter, $\quad D_{2}=D_{1}+3.2 d \mathrm{~mm}$
$E(13.3) P 210$

## Check for bolt

Check for induced shear stress in bolt

$$
\begin{array}{ll}
T=\frac{\tau_{b_{(\text {ind })}} \pi i d^{2} D_{2}}{8} & \boldsymbol{E}(\mathbf{1 3 . 9}) \boldsymbol{P} \mathbf{2 1 0} \\
& \tau_{b_{(\text {ind })}}=\frac{T \times 8}{\pi i d^{2} D_{2}} \\
\text { if } \quad \tau_{b_{(\text {ind })}}<\tau_{b_{(\text {allow })},} \quad \text { Design is Safe. }
\end{array}
$$

If the induced shear stress $\tau_{b_{(\text {ind })}}$ in the bolt are less than this allowable (permissible) shear stress $\tau_{b_{(a l l o w)}}$, the design of key is safe. Otherwise increase the bolt dimensions and check once again.

## v). Flange design

Outer diameter of flange $\quad D_{3}=D_{1}+6 d \mathrm{~mm}$

$$
E(13.4) P 209
$$

The flange thickness

$$
t=0.35 D+9 \mathrm{~mm}
$$

$E(13.7) P 210$

## Check for flange

$$
\begin{aligned}
& T=\frac{\tau_{f_{(\text {ind })}} \pi D_{1}^{2} t}{2} \quad \boldsymbol{E}(\mathbf{1 3 . 1 1 ) P} \mathbf{2 1 0} \\
& \tau_{f_{(\text {ind })}}=\frac{2 \times T}{\pi{D_{1}}^{2} t} \\
& \text { if } \quad \tau_{f_{(\text {ind })}}<\tau_{f_{(\text {allow })}, \quad \text { Design is Safe. }}
\end{aligned}
$$

If the induced shear stress $\tau_{f_{(\text {ind })}}$ in the flange are less than this allowable (permissible) shear stress $\tau_{f_{(\text {allow })}}$, the design of key is safe. Otherwise increase the flange dimensions and check once again.

|  | Symbols (Flexible (Bush Pin) Protected) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Formula | Equations | Table |
| (i). Design of Shaft |  |  |  |  |
| T | Torque transmitted in N mm | $\begin{aligned} & \frac{9.55 \times 10^{6}(P)}{n} \\ & P \text { in } \mathrm{kW} \end{aligned}$ | (3.3a) P-42 |  |
| $\eta$ | Key way factor | Assume$\eta=0.75 \mathrm{~mm}$ |  |  |
| D | Shaft Diameter | $\tau=\frac{16 T}{\pi \eta D^{3}}$ | (3.1)P-48 | (3.5a)P-48 |
|  |  | $\theta=\frac{584 T L}{G D^{4}}$ | (3.2)P-48 |  |

## Design of Key

| $b$ | Width of taper <br> key |  |  | (4.2) P-61 |
| :--- | :--- | :--- | :--- | :--- |
| $h$ | Height of taper <br> key |  | (4.2) P-61 |  |
| L | Length of taper <br> key $\approx$ Length of <br> Hub | $\mathrm{L}=1.2 \mathrm{D}+20 \mathrm{~mm}$ | (13.6) P-210 |  |

## Check for key

| $\sigma_{k_{\text {(induced }}}$ | Induced crushing <br> stress in key | $T=\frac{1}{4} \sigma_{k_{\text {(ind) }} h d L}$ | $(4.5 \mathrm{~b}) \mathrm{P}-53$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $\tau_{k_{\text {(induced }}}$ | Induced <br> stress in key | $T=\frac{1}{2} \tau_{k_{(\text {ind })} b D L}$ | (4.6) P-54 |  |

## Design of Hub

| $D_{1}$ | hub diameter | $D_{1}$ <br> $=1.8 D+20 \mathrm{~mm}$ | $(13.2) \mathrm{P}-209$ |  |
| :--- | :--- | :--- | :--- | :--- |
| L | hub length | $\mathrm{L}=1.2 \mathrm{D}+20 \mathrm{~mm}$ | $(13.6) \mathrm{P}-210$ |  |

## Design of Bolt

| No of bolts pins | i | $\begin{aligned} & i=\frac{1}{40} D+2 \\ & \text { to } \frac{3}{80} D+2 \end{aligned}$ | (13.1) P-209 |  |
| :---: | :---: | :---: | :---: | :---: |
| d | Minor Diameter of bolt | $\begin{aligned} & d \\ & =\frac{0.423 D}{\sqrt{i}} \\ & +7.5 \mathrm{~mm} \end{aligned}$ | (13.5) P-210 | (9.8)P-113 |
| $d_{p}$ | major of bolt | Assume | $d_{p}=2 \times d$ |  |
| $t_{b}$ | Thickness of brass bush | Take | $t_{b}=3 \mathrm{~mm}$ |  |
| $t_{r}$ | Thickness of rubber bush | Take | $t_{r}=6 \mathrm{~mm}$ |  |
| $d_{p i}$ | Inside diameter of rubber bush |  | $d_{p}+2 t_{b}$ |  |
| $d^{\prime}$ | Inside diameter of rubber bush |  | $d_{p i}+t_{r}$ |  |
| $D_{2}$ | Bolt circle diameter | $\begin{aligned} & D_{2} \\ & =D_{1}+3.2 \mathrm{dmm} \end{aligned}$ | (13.3) P-209 |  |
| F | Force on each pin |  | $i F\left(\frac{D_{2}}{2}\right)$ |  |
| F | Force on each pin |  | $=P_{p} l d^{\prime}$ |  |


| Check for bolt |  |  |
| :---: | :---: | :---: |
| $\tau_{p}$ | shear stress in pin | $\tau_{p}=\frac{F}{\frac{\pi}{4} d_{p}{ }^{2}}$ |
| C | Gap between flange | Take $\quad \mathrm{C}=5 \mathrm{~mm}$ |
| $\sigma_{b}$ | Bending stress in pin | $\sigma_{b}=\frac{F\left(\frac{l}{2}+C\right)}{\frac{\pi}{32} d_{p}{ }^{3}}$ |
| $\sigma_{\text {Max }}$ | Maximum normal stress in pin | $\begin{aligned} & \sigma_{M a x}=\sigma_{1} \\ & =\frac{\sigma_{b}}{2} \\ & +\sqrt{\left(\frac{\sigma_{b}}{2}\right)^{2}+\tau_{p}{ }^{2}} \end{aligned}$ |
| $\tau_{\text {Max }}$ | Maximum shear stress in pin | $\begin{aligned} & \tau_{\operatorname{Max}} \\ & =\sqrt{\left(\frac{\sigma_{b}}{2}\right)^{2}+\tau_{p}^{2}} \end{aligned}$ |

## Design of flange

\(\left.$$
\begin{array}{|l|l|l|l|l|}\hline D_{3} & \begin{array}{l}\text { Outer diameter } \\
\text { of flange }\end{array}
$$ \& \begin{array}{l}D_{3} <br>

=D_{1}+6 d \mathrm{~mm}\end{array} \& (13.4) \mathrm{P}-209\end{array}\right]\)| (13.7) P-210 |
| :--- |

## Check for flange

| $\tau_{f_{\text {(induced) }}}$ | Induced shear <br> stress in Flange | $T$ <br> $=\frac{\tau_{f_{\text {(ind })}} \pi}{} D_{1}^{2} t$ <br> $t_{1}$ | Thickness of <br> protective <br> circumferential <br> flange | Assume <br> $t_{1}=0.25 \mathrm{D}$ |
| :---: | :--- | :--- | :--- | :--- |

## Module - 4

## Riveted Joints and Weld Joints:

Riveted Joints: Rivet types, rivet materials, failures of riveted joints, Joint Efficiency, Boiler Joints, Lozanze Joints, Riveted Brackets, eccentrically loaded joints.
Weld Joints: Types of welded joints, Strength of butt and fillet welds, welded brackets with transverse and parallel fillet welds, eccentrically loaded welded joints.

## RIVETED JOINTS

## NUMERICAL PROBLEMS:

1. Explain with neat sketch, Failures of riveted joints.
( June/July 2016) (Dec. 14 / Jan. 15) (Marks 12)
2. Design a double riveted lap joint for joining two plates of thickness 10 mm . allowable stresses are $\sigma_{t}=60 \mathrm{MPa}, \sigma_{c}=80 \mathrm{MPa}$ and $\tau=50 \mathrm{MPa}$
(IP June/July 2013) (Marks 10)
3. Design a double riveted lap joint with chain riveting for mild steel plates of 20 mm thickness. allowable stresses are $\sigma_{t}=90 \mathrm{MPa}, \sigma_{c}=120 \mathrm{MPa}$ and

4. A double riveting lap joint is to be made between 9 mm plates. If the safe working stress in tension, crushing and shear are $80 \mathrm{MPa}, 120 \mathrm{MPa}$ and 60 MPa respectively, design the riveting joint.
(IP Dec 2012) (Marks 14)
5. Design a double riveted lap joint with zig - zag riveting for 13 mm thick plates. The working stresses to be used are $\sigma_{t}=80 \mathrm{MPa}, \sigma_{c}=120 \mathrm{MPa}$ and $\tau=$ 60 MPa . state how the joint will fail and find the efficiency of the joint.
(Dec. 16 / Jan. 17) (ME June/July 2011) (Marks 10)
6. Design a double riveted butt joint to join two plates of thickness 10 mm . The allowable stresses for plate material in tension are equal to 120 MPa , in compression 160 MPa , in shear 80 MPa . The widths of cover plates are equal.
(Dec. 16 / Jan. 17) (IP May / June 2010) (AU July 2006) (Marks 14)
7. Design a double riveted butt joint to connect two plates 20 mm thick. The joint is zig - zag riveted and has equal width cover plates. The allowable tensile stress for the plate is 100 MPa . The allowable shear and crushing stresses for rivet material are 60 MPa and 120 MPa respectively. Calculate the efficiency of the joint should be leak proof.
(AU Dec. 07 / Jan. 08) (Marks 10)

## Boiler joints:

1. Design a double riveted butt joint, with single cover plate, for the longitudinal seam for a boiler shell of diameter 1000 mm and pressure 1.5 MPa . Allowable tensile stress for the plate is 100 MPa and allowable shear and crushing stresses for rivets are 70 and 150 MPa respectively.
(Dec. 2010)(15 M)
2. Design a double revised double cover butt joint for the longitudinal seam of a boiler of diameter 1.2 m and for a steam pressure of 2.4 MPa . The following stresses may be used. Allowable tensile stress $=90 \mathrm{MPa}$, allowable shear stress $=60 \mathrm{MPa}$ and allowable crushing stress $=150 \mathrm{MPa}$. Assume a joint efficiency of $80 \%$.
(June/July 2014) (Marks 10)
3. Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.9 MPa . Assume joint efficiency as $75 \%$. Allowable stress in tension as 83 MPa , in compression 138 MPa and shear stress in rivets may be assumed as 55 MPa . Assume chain riveted joint.
(ME/AU June 2012) (Marks 10)
4. Design a longitudinal double riveted butt joint with two cover plates for boiler shell of 1500 mm in diameter subjected to an internal pressure of 0.915 MPa . Assume joint efficiency as $75 \%$. Take the Allowable stress in tension as $84 M P a$, in compression 140 MPa and shear stress in rivets may be assumed as $56 M P a$
(AU June/July 2008) (Marks 10)
5. Design a double riveted butt joint with two cover plates for longitudinal seam of a boiler shell of 1.5 m in diameter subjected to an internal pressure of 0.95 MPa . Assume joint efficiency as $75 \%$. Take the Allowable stress in tension as 90 MPa , in compression 140 MPa and shear stress in rivets may be assumed as 56 MPa (ME Dec. 13 / Jan. 14) (ME Dec. 13 / Jan. 14) (AU Dec. 09 / Jan. 10) (Marks 10)
6. Deign a double riveted butt joint (Staggered) with cover plate for longitudinal seam of boiler shell 1.5 m in diameter subjected to a steam pressure of 1 MPa . Assume joint efficiency as $80 \%$, allowable tensile stress in plate $=90 \mathrm{MPa}$, compressive stress $=140 \mathrm{MPa}$ and shear stress $=56 \mathrm{MPa}$.
(IP June/July 2011) (Marks 14)
7. Design a longitudinal double riveted double strap butt joint with unequal straps for a pressure vessel. The internal diameter of the pressure vessel is 1 m and is subjected to an internal pressure of 2.2 MPa . The pitch of the rivet in the outer is to double the pitch in the inner row. The allowable tensile stress in the plate is 124 MPa . The allowable shear stress and crushing of the rivets are 93 MPa and $165 M P a$ respectively. The resistance of the rivets in double shear is to be taken as 1.875 times that of single shear.
(May 2017)(ME Dec. 09 / Jan. 10) (Marks 10)
8. Design the longitudinal steam joint of a steam drum whose inner diameter is 1680 mm and the pressure of steam is 2.1 MPa by gauge. The longitudinal joint is triple riveted butt joint with an efficiency of $85 \%$. The pitch of the outer row rivets is to be double of that in the inner rows and the widths of the cover plates are unequal. The ultimate tensile, crushing and shear stresses are $470 \mathrm{MPa}, 780 \mathrm{MPa}$ and 390 MPa respectively. Adopt a factor of safety of 5. The rivet in double shear is to be greater than $87.5 \%$ over that in single shear.
(Dec. 2011)(10 M)
9. A cylindrical pressure vessel with 1 m inner diameter is subjected to internal steam pressure of 1.5 MPa the permissible stress for the cylinder plate and rivets in tension, shear and compression are 80,60 and 120 MPa respectively the efficiency of longitudinal joint can be taken as $80 \%$ for the purpose of calculating the plate thickness. The efficiency of circumferential lap joint stated to be at least $62 \%$. Design the circumferential lap joint and calculate : i) Thickness of plate; ii) Diameter of rivets ;iii) Pitch ; iv) Number of rivets ; v) Number of rows of rivets.
(IP June/July 2013) (Marks 10)
10. Design triple riveted butt joint with double straps of equal width longitudinal butt joint for a boiler shell of 1.5 m diameter. The maximum steam pressure in the boiler is limited to 2.4 MPa . The rivet pitch is to be same in all rows and chain riveting is to be used. The allowable stresses in tension, shear and crushing are $124 M P a, 93 M P a$ and $165 M P a$ respectively. Assume that the rivets in double shear are 1.875 times stronger than in single shear. Take the corrosion allowance in thickness of plate as 1 mm , sketch the joint with all the dimensions.
(AU Dec. 06 / Jan. 07) (Marks 10)
11. Design the longitudinal joint for a 1.25 m diameter steam boiler to carry a steam pressure of 2.5 MPa . The ultimate strength of the boiler plate may be assumed as 420 MPa , crushing strength of 650 MPa and shear strength of rivets as $300 M P a$. Take joint efficiency as $80 \%$. Use a factor of safety of 5 .
(IP June 2012) (Marks 12)
12. Design a triple riveted longitudinal double strap butt joint with unequal strap for a boiler. The inside diameter of the longest course of the drum is 1.3 m . The joint is to be designed for a steam pressure of 2.4 MPa. The working stresses to be used are $\sigma_{t}=77 \mathrm{MPa}$ for plate material in tension, $\tau=62 \mathrm{MPa}$ for rivet material in shear, $\sigma_{c}=120 \mathrm{MPa}$ for rivet material in compression. Assume joint efficiency as $81 \%$.
(AU July 2007) (Marks 10)
13. Design a triple riveted butt joint with two unequal cover plates, for a 1.25 m diameter steam boiler, to carry steam pressure 2.5 MPa . Give the design calculations for the longitudinal joint for the following working stress for steel plated and rivets $\sigma_{t}=84 \mathrm{MPa}, \sigma_{c}=130 \mathrm{MPa}$ and $\tau=60 \mathrm{MPa}$.
(IP Dec. 2011) (Marks 14)
14. Design the longitudinal and circumferential joint for a boiler whose diameter is 1.8 meter and is subjected to an internal pressure of 2.5 MPa . Assume $\sigma_{t}=$ $120 \mathrm{MPa}, \sigma_{c}=160 \mathrm{MPa}$ and $\tau=80 \mathrm{MPa}$.
(AU Dec. 08 / Jan. 08) (Marks 08)
15. Design a triple riveted butt joint to join two plates of thickness 10 mm . The pitch of the rivet in the outer rows, which are in single shear, is twice the pitch of the rivets in the inner rows, which are in the double shear. The stresses are as follows: tearing strength $=120 \mathrm{MPa}$, shear strength $=80 \mathrm{MPa}$ \& crushing strength $=160 M P a$. Draw neat sketches of the joint in two views.
(Dec. 14 / Jan. 15) (IP Dec. 08 / Jan. 09) (IP Dec 2010) (Marks 14)
16. Design a triple riveted longitudinal double strap butt joint with unequal strap for a boiler. The inside diameter of the longest course of the drum is 1.3 m . The joint is to be designed for a steam pressure of 2.4 MPa . The working stresses to be used are $\sigma_{t}=77 \mathrm{MPa}$ for plate material in tension, $\tau=$ 62 MPa for rivet material in shear, $\sigma_{c}=120 \mathrm{MPa}$ for rivet material in compression. Assume joint efficiency as $81 \%$. The longest pitch in outer row is twice the pitch in inner row and inner rows are zig - zag.
(ME Dec. 2011) (Marks 12)
17. Design a triple riveted lap joint of zig-zag type for a pressure vessel of 1.5 m diameter. The maximum pressure inside the vessel is 1.5 MPa . Allowable stresses in tension, crushing and shear are $100 \mathrm{MPa}, 125 \mathrm{MPa}$ and 75 MPa respectively.
(June/July 2016) (Marks 10)

## Structural or Diamond or Lozenge Joints

1. The lengths of a flat tie bar, 15 mm thick, are connected by a butt joint with equal cover plates on either side. If 400 kN is acting on the tie bar, design the joint, such that the section of the bar is not reduced by more than one rivet hole. Working stresses for the material of the bar are 85 MPa in tension, 60 MPa in shear and $110 M P a$ in crushing.
(ME May / June 2010) (Marks 10)
2. Two length of a mild steel tie-rod having width 200 mm and thickness 12.5 mm are to be connected by means of a butt joint with double cover plates. Design the joint if the permissible stresses are 105 MPa in tension, 70 MPa in shear and $180 M P a$ in crushing. Assume to double shear strength 1.875 times in single shear.
(AU June/July 2009) (Marks 10)
3. Two mild steel tie bars, for a bridge structure are to be joined by means of butt joint with double cover plates. The thickness of the tie bar is 15 mm and carries a tensile load of 300 kN . Design an economical joint completely taking the allowable stresses as $\sigma_{t}=80 \mathrm{MPa}, \sigma_{c}=160 \mathrm{MPa}$ and $\tau=64 \mathrm{MPa}$. Draw neatly a proportional top and front views of the arrangement of rivets with dimensions.
(Dec. 15/ Jan. 16) (Marks 14)
4. A tie bar in a bridge consists of a plate 350 mm wide and 20 mm thick. It is connected plate of same thickness by a cover butt joint. Design an economical structural joint permissible stresses are, tensile stress 90 MPa , shear stress 60 MPa, compression stress 150 MPa .
(June/July 2009)(6 M)
5. Design a diamond lap joint for a mild steel flat tie bar $200 \mathrm{~mm} \times 10 \mathrm{~mm}$ using 21 mm diameter rivets. Number of rivets in the joint are 8. Allowable stresses are: $\sigma_{t}=120 \mathrm{MPa}, \sigma_{c}=210 \mathrm{MPa}$ and $\tau=80 \mathrm{MPa}$. Assume hole diameter is equal to the rivet diameter.
(Dec. 2012)(05 M)

## WELDING: <br> NUMERICAL PROBLEMS:

1. A plate, 60 mm wide and 12 mm thick is to be welded to another plate, by means of transverse fillet weld at the ends. If the allowable tensile stress is 100 MPa, determine the length of the weld. (IP)(Dec. 2011)( 06 Marks)
2. A plate 95 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. (1). the maximum tensile and shear stresses are $70 M P a$ and $56 M P a$ respectively, find the length of each parallel fillet weld if the joint is subjected to static loading.
(IP)(June/July 2011)( 06 Marks)
3. A 80 mm wide and 15 mm thick plate is joined with another plate by a single transverse fillet weld and a double parallel fillet welds. Determine the length of parallel fillet weld if the joint is subjected to both static and fatigue loading. Take $\sigma_{t}=90 M P a, \tau=55 M P a$ as the allowable stresses in tension and shear respectively. Take stress concentration factors as 1.5 for transverse and 2.7 for parallel fillet weld. ( June/July. 2016) (Dec. 09/Jan. 10) (IP)(Dec. 08/Jan. 09) ( 07 Marks)
4. A 80 mm wide and 12 mm thick plate is subjected to axial tensile load is welded to a vertical support by a single transverse fillet weld and a double parallel fillet weld. The maximum tensile and shear stresses in the weld are 100 MPa and 70 MPa respectively. Find the length of each parallel weld. If the joint is subjected to i). Static loading and ii). Fatigue loading.
(AU)(Dec. 06/Jan. 07)( 10 Marks)
5. A welded connection is as shown in Fig.(4). If the allowable stress is 100 MPa , determine the size of weld.


Fig.(4)
6. A bracket carrying a load of 15 kN is to be welded as shown in the Fig (5). Find the size of the weld required if the allowable shear stress is not to exceed 80 MPa. (AU)(July. 2007) (ME)( June/July. 2011) ( 10 Marks)

$$
\mathrm{P}=15 \mathrm{kN}
$$



Fig.(5)
7. A bracket as shown in the Fig (6) carries a load of 10 kN . Find the size of the weld if the allowable shear stress is not to exceed 80 MPa .
(ME) (Dec. 2011) ( 08 Marks)
$\mathrm{P}=10 \mathrm{kN}$
Fig.(6)

8. Determine the weld size for a welded joint as shown in Fig (7).
(AU)(July.2006)( 10 Marks)


Fig.(7)
9. The following Fig (8) shows connections of eccentrically loaded welded joints. The allowable shear stress in the fillet weld using M.S. bar electrodes can be taken as 80 MPa . Find thickness of plate.


Fig.(8)
(AU)( June/July. 2009)( 10 Marks)
10. Determine the size of the weld required for a flat plate, welded to a steel column and loaded as shown in Fig (9). The allowable stress in the weld is limited to $80 M P a$ at the throat section.


Fig.(9)
(ME)(Dec. 2010)( 10 Marks)
11. Determine the size of the weld required for an eccentrically loaded weld as shown in Fig (10). The allowable stress in the weld is 75 MPa .
(ME)(Dec. 2012)( 10 Marks)


Fig.(10)
12. Determine the size of the weld required for an eccentrically loaded weld as shown in Fig (11). Assume steady load and fillet weld.


Fig.(11)

## (Dec. 14/Jan. 15)

13. Determine the size of the weld for a welded joint loaded as shown in Fig (12), if the permissible shear stress for the weld material is 75 MPa .


Fig.(12)
(May 2017) ( June/July 2014)(10 M)
14. A bracket is welded to a side column as shown in $\operatorname{Fig}(12.1)$. with a permissible stress of 80 MPa . Determine the maximum load that the bracket can withstand if the size of the weld is 10 mm .
(Dec. 09/Jan. 10)( 10 Marks)


Fig.(12.1)
15. A 16 mm thick plate is welded to a vertical support by two fillet welds as shown in $\operatorname{Fig}(12.2)$. determine the size of weld, if the permissible shear stress for the weld material is 75 MPa . (May/June 2010)(08 M)


Fig.( 12.2)
16. A welded joint as shown in the Fig (13) is subjected to an eccentric load of 2 kN . Find the size of the weld, if the maximum shear stress in the weld is 25 MPa .
(IP)(June. 2012)( 08 Marks)

17. A bracket is welded to the vertical plate by means of two fillet welds as shown in the Fig (14).determine size of the weld, if the permissible shear stress is 70 MPa .
(IP)(June/July. 2013)( 10 Marks)


Fig.(14)
18. Determine the maximum normal stress and the maximum shear stress in the weld shown in the Fig (15). (ME)(June/July. 2013)( 10 Marks)


Fig.(15)
19. A shaft of rectangular cross section is welded to a support by means of fillet welds as shown in the Fig (16).determine the size of the weld if the permissible shear stress in the weld material is limited to 75 MPa .

(AU)(June/July. 2008)( 10 Marks)
20. One end of a rectangular bar of $120 \mathrm{~mm} \times 70 \mathrm{~mm}$ cross section is welded to a vertical support by four fillet welds along its circumstance. A steady transverse load of 10 kN is applied at the free end of the bar of length 160 mm and is parallel to 120 mm side. Determine the size of the weld, if the allowable stress in the material is limited to 115 MPa .

(ME) (Dec. 09/Jan. 10)( 10 Marks)
21. Determine the size of the weld as shown in the Fig (17) if the permissible shear stress in the weld material is limited to 75 MPa .

## (AU)(June/July. 2008)( 10 Marks)


22. A welded joint as shown in fig. is subjected to an eccentric load of 2 kN . Find the size of weld, if the maximum shear stress in the weld is 25 MPa .


## (May 2017)( 10 Marks)

23. A $150 \times 100 \times 10 \mathrm{~mm}$ angle is to be welded to a steel plate by fillet welds along the edges of the 150 mm leg. The angle carries a load of 20 kN . Determine the weld length required if the permissible shear stress in the weld material is 75 MPa . The line of action of the load coincide with the gravity axis of thee section.
(Dec. 16/Jan. 17)( 12 Marks)
24. A $125 \times 100 \times 10 \mathrm{~mm}$ unequal leg angle section is to be welded to a steel plate by fillet welds along the edges of the 125 mm leg as shown in Fig 18. The angle is subjected to a tensile load of 100 kN passing through the center of gravity of angle.
(Dec. 09/Jan. 10)( 10 Marks)
25. Determine the weld lengths if the size of the weld is 8 mm and allowable shear stress in the weld is 102 MPa . All dimensions in the figure are in mm .

26. A solid circular bar of diameter 60 mm is to be welded to a vertical plate by an all round fillet weld. It carries a load of 10 kN at a distance of 200 mm from the vertical plate. Determine the size of the weld, if the maximum permissible stress is 100 MPa .

## Riveted joints

Riveted joint is permanent joint that connote be dismantled without destruction of the rivets. Tanks, pressure vessels, bridges and building structures commonly built of steel plates rolled shapes are riveted together. Rivets can be used as pivot.

## Types of riveted joints

Riveted joints are classified into two types

1. Lap joint
2. Butt joint

In the lap joints the plates to be joined overlap each other to sufficient amount for riveting. If there is a single row of rivets, the joint is called single riveted lap joint. If there are two rows of rivets, the joint is called double riveted lap joint and so on. The rivets may be arranged in chain form or zig - zag form.

When the two plates are placed end to end and are connected by cover plates ( butt straps), they form a butt joint. Normally two cover plates are used and are of equal width, but sometimes the outer strap may be narrow than the inner one. Butt joints are called single riveted, double riveted, triple riveted, etc., depending upon the number of rows of rivets in each main plates.

## Terminology

1. Pitch (p): It is the distance between the centers of two consecutive rivets in row
2. Margin (m): It is the distance from end of plate to first row of rivets.
3. Transverse pitch $\left(\mathbf{p}_{\mathbf{t}}\right)$ : The distance between two consecutive rows of rivets is called transverse pitch.
4. Diagonal pitch ( $\mathbf{p}_{\mathbf{d}}$ ): In zig - zag riveting, the distance between centers of adjacent rivets in side by side rows is diagonal pitch.

## ***Explain with neat sketch, type of Failures of riveted joints

## Failures of riveted joints

A riveted joint may fail in one of the following ways:
i). shearing of rivets:
(Fig. 1.1 )

If the diameter of the rivet is smaller than the necessary standard diameter, this type of failures will occur.
ii). Tearing of plates across the row of rivets: (Fig. 1.2 )

This type of failure will occur, if the rivets are very close each other. This can be avoided by using standard pitch.


Fig. 1.2
iii). Tearing of the margin: (Fig. 1.3)

If the hole of is too close to the edge of plate then this type of failure will occur. This can be avoided by using standard margin.

iv). Crushing of the rivets and rivet holes:
(Fig. 1.4 )
v).Shearing of margin: (Fig. 1.5)
vi). Rupturing of the pate by tension in zig-zag line passing diagonally between the rivet holes in staggered riveting.

## Type of riveted joints:



Lap joint with chain riveting

(b) Double riveted
(b) Triple riveted

Lap joint with chain riveting


Cover Plate

(a) Double riveted single cover butt joint with zig - zag riveting

(a) Double riveted

(b) Triple riveted

Double cover butt joint with zig - zag riveting

## Design procedure for riveted joins to connect two plates of thickness ' $\mathbf{t}$ '

Step 1: Thickness of main plate, $t$ (is given)
Diameter of the rivet, Unwin's empirical
formula:

$$
d=6.07 \sqrt{t} \text { to } 6.325 \sqrt{t} \quad \ldots . . \boldsymbol{E} 5.22 c(P-64)
$$

Find ' $d$ ' and select standard diameter, $d$ of the rivet from Table 5.3b (P-68A)
Step 2: Pitch (p)
a) Pitch in general case, considering the strength of perforated plate $=$ shear strength of the rivets,

$$
p=\frac{\left(n_{1}+1.875 n_{2}\right) \pi d^{2} \tau}{4 t \sigma_{t}}+d \quad \boldsymbol{E} 5.23 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 5})
$$

b) As per IBR, $p=k_{1} t+41 \mathrm{~mm} \quad \boldsymbol{E} 5.23 \boldsymbol{b}(\boldsymbol{P}-65)$

Where $k_{1}$ is constant factor from boiler code (Table 5.4a) (P-68A)
Take the smaller value of pitch as standard pitch from above two values.
Step 3: Transverse pitch,
Equal rivets in each row, For chain riveting,

$$
p_{t} \geq 2 d \quad \boldsymbol{E} 5.31 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 5})
$$

For zig-zag (staggered) riveting,

$$
p_{t} \geq 0.33 p+0.67 d \quad \boldsymbol{E} 5.31 \boldsymbol{b}(\boldsymbol{P}-\mathbf{6 5})
$$

Step 4: Margin $\quad m=1.5 d \quad E 5.34(P-66)$
Step 5: Thickness of the cover plates
i). For lap joint, no cover plates.
ii). For single cover butt joint, thickness of the cover plate,

$$
t_{i}=1.125 t \quad \boldsymbol{E} 5.4(\boldsymbol{P}-\mathbf{6 2})
$$

iii). For double cover butt joint, with equal cover plates,

$$
t_{i}=t_{o}=0.625 t \quad \boldsymbol{E} 5.6(\boldsymbol{P}-\mathbf{6 2})
$$

iv). For double cover butt joint, with unequal cover plates
a. Thickness of the narrow plates,

$$
t_{o}=0.625 t \quad \boldsymbol{E} 5.8 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 3})
$$

b. Thickness of the wider plates,

$$
t_{i}=0.750 t \quad \boldsymbol{E} 5.8 \boldsymbol{b}(\boldsymbol{P}-\mathbf{6 3})
$$

Step 6: Length of shank of the rivet,

$$
L=\sum t+(1.5 \text { to } 1.7) d \quad \boldsymbol{E} 5.9(\boldsymbol{P}-\mathbf{6 3})
$$

For lap joint,

$$
L=2 t+1.5 d
$$

For single strap butt joint,

$$
L=t+t_{i}+t_{o}+1.5 d
$$

For double strap butt joint,

$$
L=t+t_{i}+1.5 d
$$

Step 7: Strength calculation

Calculate the strength of solid plate ' $P$ ' for one pitch length,

$$
P=p t \sigma_{t} \quad \boldsymbol{E} 5.10(\boldsymbol{P}-\mathbf{6 3})
$$

Determine the least resistance of the joint by considering all possible cases of failure.

Calculate the tensile strength of solid plate ' $P_{t}$ ' for one pitch length,

$$
P_{t}=(p-d) t \sigma_{t} \quad \boldsymbol{E} 5.11(\boldsymbol{P}-63)
$$

Calculate the shear strength of solid plate ' $P_{S}$ ' for one pitch length,

$$
P_{s}=\left(n_{1}+1.875 n_{2}\right) \frac{\pi d^{2}}{4} \tau \quad \boldsymbol{E} 5.13(\boldsymbol{P}-\mathbf{6 3})
$$

Calculate the crushing strength of solid plate ' $P_{c}$ ' for one pitch length,

$$
P_{c}=\left(n_{1} t_{1}+n_{2} t\right) d \sigma_{c} \quad \boldsymbol{E} 5.14(\boldsymbol{P}-63)
$$

Step 8:
Efficiency of joints,

$$
\eta=\frac{\text { Least strength of the joint }}{\text { Strenght of solid plate }}
$$

This efficiency must be within the range given in table (T-5.1)(P-68). If the efficiency thus determined happens to be lower than the range of values given in the table, it should be increased by changing the pitch or the diameter of the rivets or by changing their arrangement of rivets.

## Riveted joints for boilers and pressure vessels

A pressure vessel stores pressurized fluid in the cylinder. If the fluid is steam, then it is known as boiler. A pressure vessel or boiler has two joints -

1. Longitudinal joints
2. Circumferential joint.

Below figure shows the riveted joints used for boilers.
Symbols: $\mathrm{D}=$ inside diameter of the boiler or pressure vessel, mm .
$p_{i}=$ fluid pressure or steam pressure, MPa .

## Design procedure of boiler joints

## Part - I: Longitudinal joint

Step 1: Select the type of joint from Table (T-5.2)(P-68) for the given diameter of the boiler.

Step 2: Thickness of plate, $\quad t=\frac{p_{i} D}{2 \sigma_{t} \eta} \quad \ldots . . E 5.1(P-62)$

$$
\eta=\text { Efficiency of joint } \cong 0.7 \text { to } 0.9(\cong 0.8) \text { Table } 5.1(\boldsymbol{P}-\mathbf{6 8})
$$

Step 3: Diameter of the rivet, Unwin's empirical formula:

$$
d=6.07 \sqrt{t} \text { to } 6.325 \sqrt{t} \quad \ldots . . \boldsymbol{E} \mathbf{5 . 2 2 c}(\boldsymbol{P}-\mathbf{6 4})
$$

Find ' $d$ ' and select standard diameter, $d$ of the rivet from Table 5.3b (P-68A)
Step 4: Pitch (p)
a) Pitch in general case , considering the strength of perforated plate $=$ shear strength of the rivets,

$$
p=\frac{\left(n_{1}+1.875 n_{2}\right) \pi d^{2} \tau}{4 t \sigma_{t}}+d \quad \boldsymbol{E} 5.23 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 5})
$$

b) As per IBR, $p=k_{1} t+41 \mathrm{~mm} \quad \boldsymbol{E} 5.23 \boldsymbol{b}(\boldsymbol{P}-\mathbf{6 5})$

Where $k_{1}$ is constant factor from boiler code (Table 5.4a) (P-68A)
Take the smaller value of pitch as standard pitch from above two values.

Step 5: Transverse pitch,
Equal rivets in each row, For chain riveting,

$$
p_{t} \geq 2 d \quad \boldsymbol{E} 5.31 a(P-65)
$$

For zig-zag (staggered) riveting,

$$
p_{t} \geq 0.33 p+0.67 d \quad \boldsymbol{E} 5.31 \boldsymbol{b}(\boldsymbol{P}-\mathbf{6 5})
$$

Step 6: Margin $\quad m=1.5 d \quad \boldsymbol{E} 5.34(\boldsymbol{P}-66)$
Step 7: Thickness of the cover plates
i). For lap joint, no cover plates.
ii). For single cover butt joint, thickness of the cover plate

$$
t_{i}=1.125 t \quad \boldsymbol{E} 5.4(\boldsymbol{P}-\mathbf{6 2})
$$

iii). For double cover butt joint, with equal cover plates

$$
t_{i}=t_{o}=0.625 t \quad \boldsymbol{E} 5.6(\boldsymbol{P}-\mathbf{6 2})
$$

iv). For double cover butt joint, with unequal cover plates
a. Thickness of the narrow plates, $t_{o}=0.625 t \quad E 5.8 a(P-63)$
b. Thickness of the wider plates, $t_{i}=0.750 t \quad \boldsymbol{E} 5.8 b(P-63)$

Step 8: Length of shank of the rivet,

$$
L=\sum t+(1.5 \text { to 1.7)d } \quad \boldsymbol{E} 5.9(\boldsymbol{P}-\mathbf{6 3})
$$

For lap joint, $L=2 t+1.5 d$
For single strap butt joint, $\quad L=t+t_{i}+t_{o}+1.5 d$
For double strap butt joint, $\quad L=t+t_{i}+1.5 d$

## Step 9: $\quad$ Strength calculation

Calculate the strength of solid plate ' $P$ ' for one pitch length,

$$
P=p t \sigma_{t} \quad \boldsymbol{E} 5.10(\boldsymbol{P}-\mathbf{6 3})
$$

Determine the least resistance of the joint by considering all possible cases of failure.
Calculate the tensile strength of solid plate ' $P_{t}$ ' for one pitch length,

$$
P_{t}=(p-d) t \sigma_{t} \quad \boldsymbol{E} 5.11(\boldsymbol{P}-63)
$$

Calculate the shear strength of solid plate ' $P_{s}$ ' for one pitch length,

$$
P_{s}=\left(n_{1}+1.875 n_{2}\right) \frac{\pi d^{2}}{4} \tau \quad E 5.13(\boldsymbol{P}-63)
$$

Calculate the crushing strength of solid plate ' $P_{c}$ ' for one pitch length,

$$
P_{c}=\left(n_{1} t_{1}+n_{2} t\right) d \sigma_{c} \quad \boldsymbol{E} 5.13(P-63)
$$

$$
E 5.16 a, 5.17 b \& 5.18 a(P-63 \& P-64)
$$

$\therefore$ Least of the above 6 values. ( 3 from part i and 3 from part ii)

Step 10:
Efficiency of joints,

$$
\eta=\frac{\text { Least strength of the joint }}{\text { Strenght of solid plate }}
$$

This efficiency must be within the range given in table (T-5.1)(P-68). If the efficiency thus determined happens to be lower than the range of values given in the table, it should be increased by changing the pitch or the diameter of the rivets or by changing their arrangement of rivets.

## Part - II Circumferential joint

Select double riveted lap joint for circumferential joint $\mathrm{d}, \mathrm{t}, p_{t}$ and m are same as part - I.

Step 1: Total steam load $\quad F=\left(\frac{\pi}{4} D^{2}\right) p_{i}$
Step 2: Strength of each rivet
i). In shear $F_{\tau}=\left(\frac{\pi}{4} d^{2}\right) \tau$
ii). In crushing $\quad F_{c}=d t \sigma_{c}$
$\therefore$ Minimum strength of the rivet $=F_{1}=$ Least of these two
Step 3: Number of rivets required, $i=\frac{\text { Steam load }}{\text { Minimum strength of the rivet }}=\frac{F}{F_{1}}$
Step 4: Rivets $/$ Row $=i /$ row $=\frac{\text { Total no.of rivets }}{\text { No.of rows of rivets }}$
Step 5: Pitch (p)
a) Pitch in general case, considering the strength of perforated plate $=$ shear strength of the rivets,

$$
p=\frac{\left(n_{1}+1.875 n_{2}\right) \pi d^{2} \tau}{4 t \sigma_{t}}+d \quad \boldsymbol{E} 5.23 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 5})
$$

b) As per IBR, $p=k_{1} t+41 \mathrm{~mm}$

E5.23 b (P-65)
Where $k_{1}$ is constant factor from boiler code (Table 5.4a) (P-68A)
Take the smaller value of pitch as standard pitch from above two values.

Step 6:check for the rivets
Also number of rivets in one row, $i=\frac{\pi(D-t)}{p}$
If the total number of rivets calculated is greater than the minimum number of rivets required, then it is safe.

## Step 7: Strength calculation

Calculate the strength of solid plate ' $P$ ' for one pitch length,

$$
P=p t \sigma_{t} \quad \boldsymbol{E} 5.10(P-\mathbf{6 3})
$$

Determine the least resistance of the joint by considering all possible cases of failure.
i). Calculate the tensile strength of solid plate ' $P_{t}$ ' for one pitch length,

$$
P_{t}=(p-d) t \sigma_{t} \quad \text { E } 5.11(P-63)
$$

ii). Calculate the shear strength of solid plate ' $P_{s}$ ' for one pitch length,

$$
P_{s}=\left(n_{1}+1.875 n_{2}\right) \frac{\pi d^{2}}{4} \tau \quad \boldsymbol{E} 5.13(\boldsymbol{P}-\mathbf{6 3})
$$

iii). Calculate the crushing strength of solid plate ' $P_{c}$ ' for one pitch length,

$$
P_{c}=\left(n_{1} t_{1}+n_{2} t\right) d \sigma_{c} \quad \boldsymbol{E} 5.13(\boldsymbol{P}-\mathbf{6 3})
$$

iv). The resistance against tearing at the plate at inner row \& shearing of the rivets in the outer row,

$$
P_{t s}=(p-2 d) t \sigma_{t}+k \frac{\pi d^{2}}{4} \tau \quad \boldsymbol{E} 5.16 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 3})
$$

Where

$$
\mathrm{k}=1 \text { for rivets in single shear }
$$

$$
\mathrm{k}=1.875 \text { for rivets in double shear }
$$

v).The resistance against tearing at the plate at inner row \& crushing of the rivets in the outer row,

$$
P_{t c}=(p-2 d) t \sigma_{t}+d t \sigma_{c} \quad \boldsymbol{E} 5.17 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 4})
$$

vi). The resistance against shearing of the rivets in the outer row \& crushing at the plate at inner row,

$$
P_{s c}=\frac{\pi d^{2}}{4} \tau+n d t \sigma_{c} \quad \boldsymbol{E} 5.18 \boldsymbol{a}(\boldsymbol{P}-\mathbf{6 4})
$$

$\therefore$ Least of the above 6 values. ( 3 from part i and 3 from part ii)
Step 8:
Efficiency of joints,

$$
\eta=\frac{\text { Least strength of the joint }}{\text { Strenght of solid plate }}
$$

This efficiency must be with in the range given in table (T-5.1)(P-68). If the efficiency thus determined happens to be lower than the range of values given in the table, it should be increased by changing the pitch or the diameter of the rivets or by changing their arrangement of rivets.

## Structural joints

Machine frames, building structures, ordinary tanks, coal bunkers, roof work, bridges etc., leakage is of minor importance. In these joints strength and rigidity are the main requirements, therefore caulking of joint is not done. In structural joints cold riveting is used and the rivet hole does not fill the hole completely. A typical type of structural joint is called as Diamond or Lozenge joint. In this joint the rivets have been arranged in such a manner that there is only one rivet in the outermost row. This makes the joint more balanced and the joint is sometimes called as economical joint or joint of uniform strength. Since in structural joints the rivets are driven cold, the rivet diameter is used for strength calculations of shear and crushing and the rivet hole diameter is used in calculating tearing strength of plate.

| $\begin{array}{\|c} \hline \text { No. of } \\ \text { Rivets } \end{array}$ | Arrangement of Rivets |  |
| :---: | :---: | :---: |
| 1 | $\oplus$ |  |
| 2 | $\bigcirc$ |  |
| 3 | $\theta-\frac{\phi}{\phi}$ |  |
| 4 | $0-0-\phi$ |  |
| 5 | $0-0-\phi$ |  |
| 6 | $\phi \frac{\phi}{-\phi}-\phi$ |  |
| 7 | $0-0 \cdot 0$ |  |
| 8 | $\theta-\frac{0-\phi}{\theta-\frac{0}{\theta}-\phi}$ |  |
| 9 | $\theta-\theta-\theta-\phi$ |  |
| 10 | $\begin{array}{ccc} \hline & \phi & \dot{\phi} \\ 0 & \phi & \phi \\ \hline 0 & 0 & \phi \\ \hline \end{array}$ |  |

## Module - 5

## Threaded Fasteners and Power Screws:

Stresses in threaded fasteners, Effect of initial tension, Design of threaded fasteners under static loads, Design of eccentrically loaded bolted joints.

Types of power screws, efficiency and self-locking, Design of power screw, Design of screw jack: (Complete Design).

## POWER SCREW:

## THEORY QUESTIONS:

1. Derive the equation for torque required to lift the load on square threads screws.
(June / July 2016) (Dec 15 / Jan 16) (08 Marks )
2. Explain self locking and overhauling in power screw.
(Dec.16/Jan. 17) (Dec 15 / Jan 16) (June / July 2014) (Dec 2010) (May / June 2010) (Dec $08 /$ Jan 09) (Dec 2011) (04 Marks )
3. Explain overhauling of screws. What is the condition for self locking? State the applications where self - locking is essential.
(May 2017) (Dec 09 / Jan 10) (Dec 2012) (05 Marks)
4. Mention at least four application of power screw.
(June / July 2009) (04 Marks )
5. What is self locking as applicable to power screw? Relate coefficient of friction to lead angle for the above condition.
(June / July 2009) (06 Marks)
6. Derive an expression for the maximum efficiency of a threaded screw and thus show that for self locking screw the efficiency is always less than $50 \%$.

## NUMERICAL PROBLEMS:

1. The square threads of a screw jack with a specification of $80 \times 16$, with double start is to raise a load of 100 kN . The mean collar diameter is 130 mm . The coefficient of friction for the threads and the collar are respectively 0.1 and 0.12 . determine :
i. The torque required to raise the load
ii. The efficiency of the screw
iii. Whether self locking exists.
(AU)(July 2006) (15 Marks)
2. A power screw for a jack has square threads of proportion $50 \times 42 \times 8$. While the coefficient of friction at the threads is 0.1 that at the collar is 0.12 . Determine the weight that can be lifted by this jack through a human effort of 400 N through a hand lever of span 400 mm .
(ME/IP/AU)(June/July 2005) (10 Marks)
3. A single threaded power screw has a major diameter restriction of 36 mm . Design the screw, if the frictional coefficient for thread and collar are 0.13 and 0.1 respectively. Estimate the power input to rotate the screw at 1 rpm , if the load to be lifted is 5 kN .
(AU)(Dec 06 / Jan 07) (10 Marks)
4. Following data apply to the mechanists clamp. Outside diameter of the screw= 14 mm , root diameter 9.5 mm , pitch $=4 \mathrm{~mm}$ (single threads). Collar friction radius $=6 \mathrm{~mm}$. Collar friction coefficient $=0.15$, screw friction coefficient $=$ 0.15 , thread angle $=300$. Assume that the mechanist can comfortably exert a maximum force of 120 N on the handle whose radius is 130 mm . Calculate the maximum clamping force that can be developed between the jaws of the clamp and the efficiency of the clamp.
(AU)(July 2007) (10 Marks)
5. A double threaded power screw with trapezoidal ISO thread is used to raise a load of 300 kN . The nominal diameter is 100 mm and the pitch is 12 mm . The coefficient of friction is 0.15 . neglecting the collar friction, calculate,
i. Torque required to raise the load
ii. Torque required to lower the load iii. Efficiency of the screw.
(AU)(Dec 2010) (10 Marks)
6. A square threaded power screw has a nominal diameter of 30 mm and a pitch of 6 mm with double threads. The load on the screw is 6 kN and the mean diameter of the thrust collar is 40 mm . The coefficient of friction for the screw is 0.1 and the collar is 0.09 . determine :
i. Torque required to raise the screw against load
ii. Torque required to lower the screw with the load
iii. Overall efficiency
iv. Is the screw of self - locking? (June /July 2016)
(AU)(June /July 2008) (AU(Dec $07 /$ Jan 08)(ME)(Dec 12)(15 Marks)
7. The lead screw of lathe has single start trapezoidal thread of 30 mm outside Dia and 6 mm pitch. It drives a tool carriage and exerts an axial load of 1.5 kN on a thrust collar of 30 mm inside dia and 50 mm outside dia. If the lead screw rotates at 40 rpm , find the power required to drive the screw. Take the coefficient of friction for power screw as 0.14 and for collar as 0.09 .

## (AU)(Dec 08 / Jan 09) (04 Marks)

8. A machine slide weighting 4000 N is elevated by two start ACME thread 50 mm diameter, 8 mm pitch at the rate of $0.8 \mathrm{~m} / \mathrm{min}$. If the coefficient of friction is 0.14 , calculate the power of the motor to drive the slide. The end of the screw is carried on a thrust collar 40 mm inside 60 mm outside diameter. The coefficient of friction at the collar is 0.12 . Assume uniform wear.

## (AU)(Dec 2011) (10 Marks)

9. A machine slide weighting 20 kN is raised by a double start square threaded screw at the rate of $0.84 \mathrm{~m} \backslash \mathrm{~min}$. take $\mu=0.12 \& \mu_{c}=0.14$. the outside diameter of the screw is 44 mm and the pitch is 7 mm . the outside and inside diameter of the collar at the end of the screw are 58 mm and 32 mm respectively. Calculate the power required to drive slide. If the allowable shear stress in the screw is 30 MPa, is the screw strong enough to sustain load.(Dec 15 / Jan 16) ( $\mathbf{0 8}$ Marks)
10. In a hand vice, the screw has double start ACME threads of 25 mm nominal diameter and 4 mm pitch. If the length of the lever is 300 mm , the maximum force that can be applied at the end of the lever is 250 N . Determine the force with which the job is held between the jaws of the vice. Take coefficient of friction at the threads is 0.14 , angle of the thread, $2 \theta=29^{0}$. Neglect collar friction.
(AU)(Dec.09/Jan.10) (10 Marks)
11. A single thread power screw of 25 mm diameter with a pitch of 5 mm , a vertical load on the screw reaches a maximum load of 500 N . The coefficients of friction are 0.05 for the collar and 0.08 for the screw. The frictional diameter of collar is 30 mm , find the torque required to rise and lower the load. Also find the efficiency of the power screw.
(AU)(June / July 2009) (10 Marks)
12. A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN . The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and color friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 rpm . Assuming uniform wear conditions at the color and allowable bearing pressure of 5.77 MPa . Find
i. The power required to rotate the screw
ii. The stresses in the screw
iii. Number of threads of nut engagement with screw and the height of the nut.
(ME)(June /July 2013) (15 Marks)
13. A vertical two start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load 18 kN . The nut off the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 20 teeth. the mechanical efficiency of the pinion and gear wheel drive is 90 percent. The axial thrust on he screw is taken by a collar bearing 250 mm outside diameter and 100 mm inside diameter. Assuming uniform pressure conditions, find minimum diameter of pinion shaft and height of nut, when coefficient of friction for vertical screw and nut is 0.15 and that for the collar bearing is 0.20 .take $\tau=$ 56 MPa and $P_{b}=1.4 \mathrm{MPa}$.
(Dec 16 / Jan 17) (14 Marks)
14. A sluice gate weighting 600 kN raised by means of two square threaded screws. The coefficient of collar friction is 0.03 and coefficient of thread friction is 0.14 . The outer diameter of the collar is 100 mm and inner diameter is 50 mm . The gate is raised at a rate of $0.6 \mathrm{~m} / \mathrm{min}$. The permissible stress of the screws material in tension and compression is $80 M P a$ and that in shear is $60 M P a$. Design the screw and nut, check for the stresses induced. Also determine the speed of screw and power required at the motor to raise the gate, assuming an efficiency of $75 \%$ for reduction drive. The permissible bearing pressure is 15 MPa .
(ME)(Dec 2011) (14 Marks)
15. The lead screw of a lathe has single start ISO metric trapezoidal threads of 52 mm nominal diameter and 8 mm pitch. The screw is required to exert an axial force of $2 k N$ in order to drive the tool carriage during turning operation. The thrust is carried on a collar of 100 mm outer diameter and 60 mm inner diameter. The values of co-efficient of friction at the screw threads and collar are 0.15 and 0.12 respectively. The lead screw rotates at $30 \mathrm{rev} / \mathrm{min}$. calculate :
i. Power required to drive the screw
ii. The efficiency of the screw
(ME)(June 2012) (14 Marks)
16. The cutter of a broaching machine is pulled by square threaded screw of 55 mm external diameter and 10 mm pitch. The operating nut takes the axial load of 400 N on a flat surface of 60 mm and 90 mm internal and external collar diameters respectively. If the coefficient of friction is 0.15 for all contact surfaces, determine the power required to rotate the nut when the cutting speed is $6 \mathrm{~m} / \mathrm{min}$. Also find the efficiency of the screw.
(Dec.13/ Jan.14) (ME)(June /July 2011)(AU)(Jan /Feb 2006) (12 Marks)
17. A single start square thread power screw is used to raise a load of 120 kN . The screw has a mean diameter of 24 mm and four threads per 24 mm length. The mean collar diameter is 40 mm . The coefficient of friction is estimated as 0.1 for the both the thread and the collar.
i. Determine the major diameter of the screw.
ii. Estimate the screw torque required to raise the load.
iii. Estimate overall efficiency.
iv. If collar friction is eliminated, what minimum value of thread coefficient is needed to prevent the screw from overhauling?
(Dec.16/ Jan.17) (June/July 2014) (Dec 09/ Jan 10) (14 Marks)
18. The screw of shaft straightner exerts a load of 30 kN as shown in fig. the screw is square threaded of outside diameter 75 mm and 6 mm pitch. Determine:
a. Force required at the rim of 300 mm diameter hand wheel assuming the coefficient of friction for threads as 0.12 .
b. Maximum compressive stress in the screw, bearing pressure on the threads
c. maximum shear stress in threads. d. Efficiency of the straightner.

(AU)(Dec 2010) (10 Marks)
19. A triple - threaded power screw is used in a screw jack, has a nominal diameter of 50 mm , a pitch of 8 mm . The threads are square shape and the length of the nut is 48 mm . The screw jack is used to lift a load of 7.5 kN . The coefficient of friction at the threads is 0.12 and collar friction is neglected. calculate
i. The principle shear stresses in the screw rod.
ii. shear stresses in the screw rod.
iii. The transverse shear stress in the screw and nut.
iv. The bearing pressure for threads
v. State whether the screw is self-locking.
(ME)(March 2001)(June/July 2009) (06 Marks)
20. A screw jack is to lift a load of 80 kN through a height of 400 mm . Ultimate strength of screw material in tension and compression are 200 MPa and in shear it is 120 MPa . The material for the nut is phosphor bronze for which the ultimate strength is 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 MPa . Design the screw and the nut and check for the stresses. Take FoS $=2$. Assume 25 \% overload for the screw rod design.
(ME) (Dec 14/ Jan 15) (May / June 2010) (16 Marks)

## Complete Design of Screw Jack

1. Design a screw jack to lift a load of 30 kN with the following data: allowable compressive stress in screw material is 160 MPa , Coefficient of friction in threads $=0.14$, coefficient of collar friction $=0.2$, and height of lift $=150 \mathrm{~mm}$.
(ME)(Dec 2012) (15 Marks)
2. A screw jack is to lift a load of 80 kN through a height of 400 mm . Ultimate strength of screw material in tension and compression are 200 MPa and in shear it is 120 MPa . The material for the nut is phosphor bronze for which the ultimate strength is 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure been the nut and the screw is not to exceed $18 M P a$.design the screw and the nut and check for the stresses. TakeFOS $=2$ assume $25 \%$ overload for the screw rod design.
(May / June 2010) ( $\mathbf{1 5}$ Marks)
3. Design a screw jack to lift a load of 30 kN with the following data: allowable compressive stress in screw material is 160 MPa , Coefficient of friction in threads $=0.14$, coefficient of collar friction $=0.2$, and height of lift $=150 \mathrm{~mm}$.
(ME)(Dec 2012) (15 Marks)
4. Design a screw jack for a capacity of 10 kN , to lift 200 mm height. Select suitable materials and factor of safety. (Dec 08/Jan 09)(Marks 14)
5. Design completely the screw, handle and the nut of a screw jack of capacity 40 kN . The maximum lift is limited to 0.2 m . The screw and the handle are made of (45C8) steel and the nut and the cup are made of cast iron. Also find the co efficiency of the screw. Check the screw for buckling load.
(ME)(June /July 2013) (20 Marks)

## POWER SCREW

A power screw is a machine device used for converting rotary motion into translational motion and at the same time it transmits power. Typical Power screw is also called translation screw. It uses helical translator motion of the screw thread in transmit in power rather than clamping the machine components.

The main applications of power screw are as follows:
(i). To rise the load, e.g. screw - jack,
(ii). To obtain accurate motion in machinating operations, e.g. lead-screw of lathe,
(iii). To clamp a work piece, e.g. vice, and clamps, fly presses.
(iv). To load a specimen, e.g. universal testing machine, control actuators.

A power screw essentially comprises of a screw form and a meshing nut. Possible combinations of power and motion transmission are as shown in below fig

a. Screw

Rotating \&
Translating,

b. Screw Rotating Nut Translating, but Not Rotating

c. Nut Rotating
\& Translating, Screw Fixed

d. Nut Rotating but Not Translating, Screw Translating

There are three essential parts of the power screw, viz. screw, nut and a part to hold either the screw or the nut in its place.

Depending upon the holding arrangement, power screws operate in two different ways. In some cases, the screw rotates in its bearing; while the nut has axial motion the lead screw of the lathe is an example of this category. In other applications, the nut is kept stationary and the screw moves in axial direction. Screw-jack and machine vice are the examples of this category.

## What are the merits and demerits of screw joints?

## Merits

1. Power screw has large load carrying capacity. A load of 15 kN can be raised by applying an effort as small as 300 N . Therefore most of the power screws used in various applications like screw-jacks, clamps, valves and vices are manually operated.
2. Power screw gives smooth and noiseless service without any maintenance.
3. Screw joints are highly reliable in operation.
4. Simple manufacture with the possibility of maintaining high accuracy.
5. They can be made in any convenient shape and small size.
6. In addition to fastening, screws and nuts are used as power screws.
7. Power screw can be designed with self-locking property. In screw-jack application, self locking characteristic is required to prevent the load from descending on its own.

## Demerits

1. A stress concentration is available in threaded portions and hence lowering their life.
2. Power screw has very poor efficiency as low as $40 \%$. Therefore, it is not used in continuous power transmission in machine tools, with the exception of the lead screw. Power screws are mainly used for intermittent motion that is occasionally required for lifting the load or actuating the mechanism.
3. High friction in threads causes rapid wear of the screw or the nut. In case of square threads, the nut is usually made of soft material and replaced when worn out. In trapezoidal threads, a split-type of nut is used to compensate for the wear. Therefore, wear is serious problem in power screws.
4. Self-loosening properties and hence air-tight joints cannot be maintained, unless by providing some locking devices.

## Forms of threads

The varies forms of threaded profiles employed in power screw are:
i. Square thread,
ii. Modified thread,
iii. Acme thread,
iv. Buttress thread

## 1. Square Threads :

As the name suggests, these threads have their flanks at right angles to the axis. They are generally used for power transmission in machine tools. (e.g. Lead screws of lathes)

## 2. Acme Threads :

This is a variation of square threads. It is much stronger than square threads and is easy to manufacture. These threads are used in brass valves, lead screws of lathes and bench vices.

## 3. ISO Trapezoidal Threads :

These are similar to Acme threads, but are standardized by ISO.

## 4. Buttress Threads :

Buttress threads are used in transmitting power only in one direction. The force transmitted is almost parallel to the axis. It has a low frictional resistance


1. Square

Threads

2. ACME Threads

3. ISO Trapezoidal Threads $2 \theta=$

| Thread form | Thread angle (2 $\boldsymbol{\theta})$ | $\boldsymbol{\theta}$ |
| :--- | :---: | :---: |
| ISO Metric Threads | $60^{0}$ | $30^{0}$ |
| ISO Trapezoidal Threads | $30^{0}$ | $15^{0}$ |
| ACME Threads | $29^{0}$ | $14.5^{0}$ |

## TERMINOLOGY OF POWER SCREW

Typical geometry and nomenclature for thread forms is as shown in below Fig.


## Nomenclature of Screw Thread

Pitch: The pitch is defined as the distance, measured parallel to the axis of the screw, from a point on one thread to the corresponding point on the adjacent thread. It is denoted by the letter ' p '.

Nominal diameter: Nominal diameter is the largest diameter of the screw. It is also called major diameter. It is denoted by the letter ' $d$ '.

Core diameter: Core diameter is smallest diameter of the screw thread. It is also called minor diameter. It is denoted by letters ' $\boldsymbol{d}_{\mathbf{1}}$ '

Helix angle:- The helix angle is defined as the angle made by the helix of the thread with a plane perpendicular to the axis of the screw. Helix angle is related to the lead and the mean diameter of the screw. It is also called lead angle. Helix angle is denoted by ' $\alpha$ '. $\tan \alpha=\frac{\boldsymbol{l}}{\pi d_{2}} \quad E(9.22)(P-107)$

Thread angle: the angle between two adjacent flanks of a thread is called thread angle. For angle threads such as acme threads, $\mathbf{2 \theta}=\mathbf{2 9}^{\boldsymbol{0}}$ and for trapezoidal threads, $\mathbf{2 \theta}=\mathbf{3 0}^{\mathbf{0}}$ and for ISO Metric threads, $\mathbf{2 \theta}=\mathbf{6 0}^{\mathbf{0}}$.

## Multiple start threads

In a single start threads, the pitch and the lead are the same. The depth of the thread is dependent on the pitch. If for an application, one requires a large axial movement for a given rotation of the nut, the pitch has to be large. A large pitch would mean that the core diameter would be small, resulting in weakening of the screw. To overcome this problem, multi - start threads are cut on the same screw. As shown in below Fig.


1. Single start Threads Lead $=$ Pitch

2. Double start Threads Lead $=2 \times$ Pitch
3. Multi start Threads Lead $=\mathrm{nx}$ Pitch

Lead: The lead is defined as the distance, measured parallel to the axis of the screw that the nut will advance in one revolution of the screw. It is denoted by the letter ' 1 '. All threads are right threads unless otherwise specified.

## 1. State the relation between pitch and lead for single start and double start threads.

Lead, $\quad \boldsymbol{l}=\boldsymbol{n} \boldsymbol{x} \boldsymbol{p}$
Where $\mathrm{n}=$ Number of starts, $\quad \boldsymbol{p}=\boldsymbol{P i t c h}$
Hence for single start, thread. $\boldsymbol{l}=\boldsymbol{p}$ and
For double start thread. $\boldsymbol{l}=\mathbf{2 p}$

## Expression for torque

Figure 1.2 is used as basis for determining the torque required to raise or lower the load. Following conclusions can be drawn on the basis of development of thread,

1. The screw can be considered as an inclined plane with $\boldsymbol{\alpha}$ as inclination.
2. The load W always acts in vertically downward direction. When the load W is raised, it moves up the inclined plane. When the load W is lowered, it moves down the inclined plane.
3. The load W is raised or lowered by means of an imaginary force $\boldsymbol{F}_{\boldsymbol{t}}$ acting at the mean radius of the screw. The force $\boldsymbol{F}_{\boldsymbol{t}}$ multiplied by the mean radius $\left(\frac{\mathbf{d}_{\mathbf{2}}}{\mathbf{2}}\right)$ gives the torque required to raise or lower the load. Remember $\boldsymbol{F}_{\boldsymbol{t}}$ is perpendicular to load W.


Fig. 1.2 Development of Thread
We will consider two separate cases to find out the torque required to raise or lower the load in the next articles.
*** Derivation of expression for torque $T$ required to raise the load and to lower the load.

The square threaded screw with single start thread of mean diameter $\boldsymbol{d}_{\mathbf{2}}$ is considered as an inclined plane with inclination $\boldsymbol{\alpha}$ subjected to axial compressive load of W as shown in Fig. 1.3.
*** Derivation of expression for torque $\mathbf{T}$ required raising the load;
Case (1): When the load is being raised, following forces act at a point on this inclined plane:
(i) Load (W): It always acts in vertically downward direction.
(ii) Normal reaction (N): It acts perpendicular (normal) to the inclined plane.
(iii) Frictional force $\left(\boldsymbol{f}_{N}\right)$ : Frictional force acts opposite to the motion.

Since the load is moving up the inclined plane, frictional force acts along the inclined plane in downward direction.


Fig. 1.3.a Force diagran for lifting load


Fig. 1.3.b Resolution of forces

Considering the equilibrium of horizontal and vertical forces, $\boldsymbol{\Sigma} \boldsymbol{H}=\mathbf{0}$ and $\boldsymbol{\Sigma} \boldsymbol{V}=\mathbf{0}$ When

$$
\begin{gather*}
\Sigma H=0, F_{t}=(f N) \cos \alpha+N \sin \alpha \\
F_{t}=N(f \cos \alpha+\sin \alpha) \tag{1}
\end{gather*}
$$

Taking

$$
\begin{gather*}
\Sigma V=0, W+(f N) \sin \alpha=N \cos \alpha \\
W=N(\cos \alpha-f \sin \alpha) \tag{2}
\end{gather*}
$$

Diving equation (1) by equation (2),

$$
\begin{aligned}
& \frac{F_{t}}{W}=\frac{N(f \cos \alpha+\sin \alpha)}{N(\cos \alpha-\sin \alpha)}=\frac{(f \cos \alpha+\sin \alpha)}{(\cos \alpha-\mathrm{f} \sin \alpha)} \quad \text { Equn (3) } \\
& \left\{\text { i. e. }, \tan (A+B)=\left[\frac{(A+B)}{(1-A . B)}\right]\right\}
\end{aligned}
$$

$$
\begin{gathered}
F_{t}=\frac{W(f \cos \alpha+\sin \alpha)}{(\cos \alpha-f \sin \alpha)}=\frac{W(f+\tan \alpha)}{(1-f \tan \alpha)} \quad E(9.23)(P-107) \\
(f=\tan \varnothing)
\end{gathered}
$$

i.e.,

$$
F_{t}=W \tan (\emptyset+\alpha) \quad E(9.23)(P-107)
$$

and, when consider friction, $T_{n}=\frac{W d_{2}}{2} \tan (\emptyset+\alpha) E(9.24 a)(P-107)$ In the absence of friction the torque,

$$
T_{n}=\frac{W d_{2}}{2} \tan \alpha \quad E(9.24 b)(P-107)
$$

Torque necessary to overcome the collar friction,

$$
T_{c}=\frac{W f_{c} d_{c}}{2} \quad E(9.28)(P-107)
$$

Where $\boldsymbol{f}_{\boldsymbol{c}}$ is the coefficient of friction between the flat collar and its mating surface usually the frame. $\quad \therefore$ Total frictional torque, $T=\boldsymbol{T}_{\boldsymbol{n}}+\boldsymbol{T}_{\boldsymbol{c}}$

$$
\begin{array}{rll}
T & =\frac{W}{2}\left[d_{2} \tan (\emptyset+\alpha)+f_{c} d_{c}\right] & E(9.29)(P-108) \\
T & =\frac{W}{2}\left[d_{2}\left[\frac{(\tan \alpha+f)}{(1-f \tan \alpha)}\right]+f_{c} d_{c}\right] & E(9.29)(P-108)
\end{array}
$$

When the nut is given one turn, the work done by the effort $\boldsymbol{f}_{\boldsymbol{t}}$ (tangential force at the mean radius of the screw) is $\boldsymbol{f}_{\boldsymbol{t}} \boldsymbol{\pi} \boldsymbol{d}_{\mathbf{2}}$ and the work done against the load W is W x 1 .
$\therefore$ Efficiency of the screw,

$$
\eta=\frac{\text { usefull work }}{\text { work input }}=\frac{W l}{f_{t} \pi d_{2}}=\frac{W}{f_{t}} \times \frac{l}{\pi d_{2}}=\frac{\tan \alpha}{\tan (\varnothing+\alpha)}
$$

By neglecting the collar friction,
Efficiency of the screw,

$$
\eta=\frac{\tan \alpha}{\tan (\phi+\alpha)} \quad E(9.27)(P-107)
$$

By considering the collar friction,

$$
\eta=\frac{d_{2} \tan \alpha}{d_{2} \tan (\alpha+\emptyset)+f_{c} d_{c}}=\frac{d_{2} \tan \alpha}{\left[d_{2}\left(\frac{(\tan \alpha+\mathrm{f})}{(1-\mathrm{f} \tan \alpha)}\right)+\right]}
$$

Efficiency of the screw,

$$
\eta=\frac{d_{2} \tan \alpha}{d_{2} \frac{\tan \alpha+f}{1-f \tan \alpha}+f_{c} d_{c}} \quad E(9.30)(P-108)
$$

$$
\eta=\frac{1}{\pi\left[d_{2} \tan (\alpha+\emptyset)+f_{c} d_{c}\right]} \quad E(9.30)(P-108)
$$

*** Derivation of expression for torque $T$ required lower the load;
Case (2): While lowering the load, the friction force $\boldsymbol{f}_{N}$ acts upwards along the inclined surface, $\boldsymbol{F}_{\boldsymbol{t}}$ to the left.


Fig. 1.4.a Force Diagram for Lowering Load


Considering the equilibrium of horizontal and vertical forces, $\boldsymbol{\Sigma} \boldsymbol{H}=\mathbf{0}$ and $\boldsymbol{\Sigma} \boldsymbol{V}=\mathbf{0}$
When $\boldsymbol{\Sigma H}=\mathbf{0}, \boldsymbol{F}_{\boldsymbol{t}}+\boldsymbol{N} \sin \alpha=(\boldsymbol{f}) \cos \alpha$

$$
\boldsymbol{F}_{\boldsymbol{t}}=\mathbf{N}(\mathbf{f} \cos \alpha-\sin \alpha) \quad \text { Equn (4) }
$$

Taking $\boldsymbol{\Sigma} \boldsymbol{V}=\mathbf{0}, \boldsymbol{N} \boldsymbol{\operatorname { c o s }} \alpha+(\boldsymbol{f} \boldsymbol{N}) \boldsymbol{\operatorname { s i n }} \boldsymbol{\alpha}=\boldsymbol{W}$

$$
\begin{equation*}
W=N(\cos \alpha+f \sin \alpha) \tag{5}
\end{equation*}
$$

Diving equation (1) by equation (2),
$\frac{F_{t}}{W}=\frac{\mathrm{N}(\mathrm{f} \cos \alpha-\sin \alpha)}{\mathrm{N}(\cos \alpha+\mathrm{f} \sin \alpha)}=\frac{(\mathrm{f} \cos \alpha-\sin \alpha)}{(\cos \alpha+\mathrm{f} \sin \alpha)} \quad$ Equn (6)
i.e.,

$$
F_{t}=\frac{W(f \cos \alpha-\sin \alpha)}{(\cos \alpha+f \sin \alpha)}=\frac{W(f-\tan \alpha)}{(1+f \tan \alpha)} \quad E(9.25)(P-107)
$$

$$
(f=\tan \emptyset)
$$

i.e.,

$$
F_{t}=W \tan (\varnothing-\alpha) \quad E(9.25)(P-108)
$$

and, when consider friction,

$$
T_{n}=\frac{W d_{2}}{2} \tan (\emptyset-\alpha) \quad E(9.26)(P-108)
$$

Torque necessary to overcome the collar friction,

$$
T_{c}=\frac{W f_{c} d_{c}}{2} \quad \mathrm{E}(9.28)(P-107)
$$

$\therefore$ Total frictional torque, $\mathrm{T}=\boldsymbol{T}_{\boldsymbol{n}}+\boldsymbol{T}_{\boldsymbol{c}}$
For V-thread screws,
Tangential force, $\quad \boldsymbol{F}_{\boldsymbol{t}}=\mathbf{W} \frac{(\tan \alpha+\mathrm{f} \sin \theta)}{(1-\mathrm{f} \tan \alpha \sec \theta)} \quad \mathbf{E}(\mathbf{9 . 3 4})(\boldsymbol{P}-\mathbf{1 0 8})$
Total friction torque including collar friction torque,

$$
\mathrm{T}=\frac{W}{2}\left[d_{2}\left(\frac{\tan \alpha+\mathrm{f} \sec \theta)}{1-\mathrm{f} \tan \alpha \sec \theta}\right)+f_{c} d_{c}\right] \quad E(9.35)(P-108)
$$

The efficiency formula for V-thread,

$$
\eta=\frac{d_{2} \tan \alpha}{d_{2} \frac{\tan \alpha+\mathrm{fsec} \theta}{1-\mathrm{f} \tan \alpha \sec \theta}+f_{c} d_{c}} \quad \mathrm{E}(9.36)(P-108)
$$

The condition for self locking of $v$-threads without collar friction is

$$
\tan \alpha<\frac{f}{\cos \theta}
$$

The relation between the applied torque and the resisting load,

$$
T=\frac{W l}{2 \pi \eta} \quad E(9.21)(P-107)
$$

To find length of the nut,

$$
l_{n}=p n=\frac{4 W_{P}}{P_{b} \pi\left(d^{2}-D_{1}^{2}\right)} \quad E(9.33)(P-108)
$$

To find number of threads,

$$
n=\frac{4 W}{P_{b} \pi\left(d^{2}-D_{1}^{2}\right)} \quad E(9.32)(P-108)
$$

**** Explain self locking and overhauling of screw?
If $(\boldsymbol{\alpha}>\emptyset)$, Then torque required to lower the load is negative. The load itself will begin to turn the screw and descend down, unless a restraining torque is applied. This condition is called 'overhauling of screw'.

If $(\boldsymbol{\alpha}<\emptyset)$, Then the torque required to lower the load will be positive and the screw is called 'self locking screw'

The condition for overhauling of square threads with collar friction is

$$
\tan \alpha \geq\left(\frac{f d_{2}+f_{c} d_{c}}{d_{2}-f f_{c} d_{c}}\right) \quad E(9.31)(P-108)
$$

## DESIGN OF SCREW JACK

A screw jack is a portable device consisting of a screw mechanism used to raise or lower the loads to a small height. a jack is a simple and widely used device, the use of any lifting device is subject to certain hazards.

The main reasons of such accidents are as follows:
(i)The load is improperly secured on the jack.
(ii) The screw jack is over loaded.
(iii)The centre of gravity of the load is off centre with respect to the axis of the jack.
(iv)The screw jack is not placed on hard and level surface.
(v) The screw jack is used for a purpose, for which it is not designed.

Proper size, strength and stability are the essential requirements for the design of the screw jack from safety considerations.

## Design procedure for power screws

In designing power screws, the following procedure is followed.
The major components of screw jack (power screws) to be designed are
(a)Screw rod
(b) Lever and
(c)The nut

a) Screw rod: (material : steel)

Design of screw rod consist of calculation of the dimensions of major diameter or nominal diameter(d), core diameter or minor diameter ' $\boldsymbol{d}_{\mathbf{1}}$ ' and pitch ' P ' of square threads of normal series.

The threaded rod is subjected to compressive load 'W' and hence the compressive stress in the rod is given by, $\sigma_{c}=\frac{W}{A_{c}}$

Where

$$
\mathrm{W}=\text { load on the rod in Newton, } \mathrm{N} \text {. }
$$

$$
A_{c}=\text { core area of the rod in } m m^{2}=\frac{d_{1}^{2}}{4}
$$

Now,

$$
A_{c}=\frac{W}{\sigma_{c}}
$$

$\boldsymbol{\sigma}_{\boldsymbol{c}}$ is calculated by selecting suitable material for the screw rod such as C40.
The values of d, $\boldsymbol{d}_{\boldsymbol{1}}$ and P based on $\boldsymbol{A}_{\boldsymbol{c}}$ are obtained from Table (T-9.10) Page (121) for normal series.

For Fine series
For Coarse series
Table (9.09) Page (118)
Table (9.11) Page (123)
b) Design of lever or handle to operate the jack


Design of handle or lever consists of calculating the length $\boldsymbol{l}_{\boldsymbol{h}}$ and diameter of handle $\boldsymbol{d}_{\boldsymbol{h}}$.

1. The length of Handle is obtained from,

$$
T=F \times l_{h} \text { or } l_{h}=\frac{T}{F}
$$

Where

$$
\mathrm{T}=\text { torque required to raise load in } \mathrm{N}-\mathrm{mm} .
$$

And
$\mathrm{F}=$ Effort applied manually at end of handle.

$$
=200 \text { to } 300 \mathrm{~N} .
$$

Torque T is obtained from

$$
T=\frac{W}{2}\left[d_{2}\left(\frac{(\tan \alpha+f)}{(1-f \tan \alpha)}\right)+f_{c} d_{c}\right] \quad E(9.29)(P-108)
$$

Where $\mathrm{W}=$ load to be lifted in N .

For angular threads, use $\mathbf{E}(\mathbf{9 . 3 5})(\boldsymbol{P}-\mathbf{1 0 8})$ for T in which angular of thread, $\mathbf{2 \alpha}=29^{0}$ for acme thread or $\mathbf{2 \alpha}=\mathbf{3 0}{ }^{0}$ for trapezoidal threads. $\boldsymbol{d}_{\mathbf{2}}=$ mean diameter in mm ,

$$
d_{2}=\frac{d+d_{1}}{2}
$$

$$
\operatorname{Tan} \alpha=\frac{l}{\pi d_{2}} \quad E(9.22)(P-108)
$$

Where
$\boldsymbol{\alpha}=$ helix angle of thread

$$
1=\text { lead of thread }=\text { pitch } P, \quad \text { for single start thread }
$$

$\mathrm{l}=2 \mathrm{P}$, for double start thread
$1=3 \mathrm{P}$, for triple start thread
$\mathrm{f}=$ coefficient of thread friction; depend on the lubrication used

$$
\boldsymbol{f}_{\boldsymbol{c}}=\text { Coefficient of collar friction }
$$

Note: depends on the material of nut and screw (use starting friction)
Foe example for soft steel screw rod and CI nut, $\boldsymbol{f}_{\boldsymbol{c}}=0.170$

$$
\begin{aligned}
& d_{c}=\text { mean diameter of collar } \\
& \qquad=\frac{(\text { outside diameter of collar }+ \text { inside diameter of collar })}{2}
\end{aligned}
$$

If diameter $\boldsymbol{d}_{\boldsymbol{c}}$ is unknown,
Use

$$
\left.\boldsymbol{d}_{\boldsymbol{c}}=1.5 \mathrm{~d} \quad \text { (an approximation }\right)
$$

Note: after obtaining $\boldsymbol{l}_{\boldsymbol{h}}$ from $\frac{\boldsymbol{T}}{\boldsymbol{F}}$, a length of 200 to 250 mm is added for holding the lever while applying effort F .

## 2. To obtain $\boldsymbol{d}_{\boldsymbol{h}}$, the diameter of handle

The material selected is usually same as that of screw
The lever is subjected to bending due to F load hence the bending stress $\boldsymbol{\sigma}_{\boldsymbol{b}}$ in the lever is given by $\sigma_{b}=\frac{M_{b}}{Z_{b}}$

Where
$\boldsymbol{M}_{\boldsymbol{b}}$ is approximated to T

And

$$
Z_{b}=\text { section modulus }=\frac{\pi d_{h}{ }^{3}}{32}
$$

Where

$$
\boldsymbol{d}_{\boldsymbol{h}}=\text { diameter of handle in } \mathrm{mm} .
$$

## 3. Design of nut

(The material of the nut is cast iron or bronze)


Design of nut includes calculation of number of threads ( n ) and height or length of nut $\left(\boldsymbol{l}_{\boldsymbol{n}}\right)$

## a. Number of threads (n)

Number of threads in nut $=\mathbf{n}=\frac{4 W}{P_{b} \pi\left(d^{2}-D_{1}^{2}\right)} E(9.32)(P-108)$
Where $\quad \boldsymbol{P}_{\boldsymbol{b}}=$ permissible bearing pressure $\quad . .$. Table (9.4) (P-110)
Note: $\boldsymbol{P}_{\boldsymbol{b}}$ depends on material of screw and nut
For example for screw jack having steel screw and bronze nut, $\boldsymbol{P}_{\boldsymbol{b}}$ is 11.0 to 17.2 $M N / m^{2}$
(ii). Length of the nut $\left(l_{n}\right)$
$\boldsymbol{l}_{\boldsymbol{n}}$ is obtained from,

$$
l_{n}=n P \quad E(9.33)(P-108)
$$

Where

$$
\mathrm{P}=\text { pitch of the threads in } \mathrm{mm}
$$

## 4. Calculation of the efficiency ( $\boldsymbol{\eta}$ )

For square threads, $\boldsymbol{\eta}$ is calculated from equation

$$
\eta=\frac{d_{2} \tan \alpha}{d_{2} \frac{\tan \alpha+f}{1-f \tan \alpha}+f_{c} d_{c}} \quad E(9.30)(P-108)
$$

## Check for maximum shear stress

$\boldsymbol{\tau}_{\boldsymbol{m a x}}=$ maximum shear stress in screw is given by

$$
\tau_{\max }=\sqrt{\frac{\sigma^{2}}{2}+\tau^{2}} \quad E(1.12)(P-02)
$$

Where

$$
\sigma=\sigma_{c}=\frac{W}{A_{c}} \text { in } N / \mathrm{mm}^{2}, \quad \tau=\frac{T}{z_{t}}=\frac{16 T}{\pi d_{1}{ }^{3}} \text { in } N / \mathrm{mm}^{2}
$$

## Check for self locking and overhaul

(i). Condition "self locking" exist when the load raised to the required height remain at that when the lever through which the effort applied is freed. $\boldsymbol{\phi}>\boldsymbol{\alpha}$
For 'self locking',
Where
$\phi=\tan ^{-1} f$
(i). Condition "overhaul" exist when the load raised to the required height does not remain at that when the lever through which the effort applied is freed. i.e., the load is moved down on its own when the lever is freed.
For 'overhaul',

$$
\alpha>\phi
$$

## Check for buckling

The screw rod is checked for buckling when the load is raised through the required lift using Johnson's parabolic equation or Rankine's formula.

Johnson's parabolic equation for buckling load is given by

$$
F_{c r}=A_{c} \times \sigma_{y p}\left[1-\frac{\sigma_{y p}}{4 n \pi^{2} E}(l / k)^{2}\right] \quad E(1.31)(P-05)
$$

Where $\boldsymbol{F}_{\boldsymbol{c r}}=$ crippling load or buckling load in N .

$$
\mathrm{A}=\boldsymbol{A}_{\boldsymbol{c}}=\text { core area of screw rod in } \boldsymbol{m m}^{\mathbf{2}}
$$

$\sigma_{\boldsymbol{y} \boldsymbol{p}}=$ yielding stress of material of screw rod in $\mathbf{N} / \boldsymbol{m m}^{\mathbf{2}}$

$$
\mathrm{n}=\text { end fixity co-efficient. }
$$

Assume the screw rod as column fixed at one end and free at the other end.
$\mathrm{n}=0.25$
E (1.29)Page (05)
$\mathrm{E}=$ young's modulus or modulus of elasticity of the material of screw rod in $\mathrm{N} / \mathrm{mm}^{\mathbf{2}} \quad$ (Refer table 1.1)

$$
\begin{aligned}
& \boldsymbol{l}=\text { lift in } \mathrm{mm} . \\
& \mathrm{R}=\text { radius of gyration }=\frac{d_{\mathbf{1}}}{4} \text { in } \mathrm{mm} .
\end{aligned}
$$

If $\boldsymbol{F}_{\boldsymbol{c r}}>\mathrm{W}$, no buckling occurs and the design is safe
If $\boldsymbol{F}_{\boldsymbol{c r}}<\mathrm{W}$, the rod undergoes buckling and hence re-design screw with higher core area.

